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Letter to the editor

A note on optimal insurance risk control with multiple reinsurers

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ABSTRACT

This note revisits the problem discussed in Meng et al. (2016) where an optimal insurance risk control problem was considered in a diffusion approximation model with multiple reinsurers adopting variance premium principles. It was shown in Meng et al. (2016) that under a certain technical condition, a combined proportional reinsurance treaty is an optimal form in a class of plausible reinsurance treaties. From both theoretical and practical perspectives, an interesting question may be whether the combined proportional reinsurance treaty is still an optimal form in a quite considerably larger class of plausible reinsurance treaties. This note addresses this question and shows that a combined proportional reinsurance treaty is still an optimal form.

1. Introduction

Since the classical work by Borch [1] and Arrow [2] on optimal reinsurance, many aspects of generalizations have been studied in the literature. One direction of these generalizations is to consider multiple (re)-insurance parties, see, for example, [3–6]. For some related discussions and relevant literature, see [7,6]. Some recent works on generalizations to dynamic modeling settings and multiple re-insurers include, for example, [8–10]. See [9] for some related discussions and relevant literature.

The purpose of this paper is to further generalize the recent work of Meng et al. [10] by enlarging the space of reinsurance treaties with a view to presenting a scientific inquiry on optimal reinsurance policies in a more general and flexible modeling environment. Using the Lagrangian function method, we establish the optimality result that the combined proportional reinsurance treaty is still an optimal form in a quite considerably larger class of plausible reinsurance treaties than the one considered in [10]. The organization of this paper is as follows. Section 2 presents the model formulation. The main result, namely the optimality result, is presented in Section 3.

2. Model formulation

In [10], we discussed an optimal risk control problem in a diffusion approximation model with the multiple reinsurers adopting variance premium principles. That is, the surplus process of the insurance company without dividends

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satisfies

$$dX(t) = \upsilon(g_0(t, Z), g_1(t, Z), \ldots, g_m(t, Z))dt + \sigma(g_0(t, Z))dB(t), \tag{2.1}$$

where $\{B(t), t \geq 0\}$ is a standard Brownian motion on a given complete, filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$,

$$\upsilon(g_0(t, Z), g_1(t, Z), \ldots, g_m(t, Z)) = \lambda \left(\theta_0\sigma^2 - \sum_{j=1}^{m} \theta_j \mathbb{E}[(g_j(t, Z))^2]\right), \tag{2.2}$$

and

$$\sigma(g_0(t, Z)) = \sqrt{\lambda \mathbb{E}[(g_0(t, Z))^2]}, \tag{2.3}$$

where $\mathbb{E}[\cdot]$ is the expectation taken under $\mathbb{P}$; Z is a nonnegative random variable; $\sigma^2 = \mathbb{E}[Z^2]$ and $\lambda, \theta_j > 0, j = 0, 1, \ldots, m$; the expectations in (2.2) and (2.3) are taken only for the random variable Z under the measure $\mathbb{P}$. In [10], a reinsurance strategy $(g_0, g_1, \ldots, g_m)$ is called an admissible policy if it satisfies the following four conditions:

(1) for each $z \in \Re^+$ and each $j = 0, 1, \ldots, m$, $\{g_j(t, z); t \geq 0\}$ is $\{\mathcal{F}_t\}_{t\geq 0}$-predictable;
(2) for each $(t, \omega) \in [0, \infty) \times \Omega$, $g_j(t, z, \omega)$ is Borel-measurable in z;
(3) $g_j(t, z) \geq 0$, $\sum_{j=0}^{m} g_j(t, z) = z$;
(4) There is at least a pair (k, l) such that

$$\sqrt{\mathbb{E}\left[\left(\sum_{i\geq 0, i\neq k} g_i(t, Z)\right)^2\right]} - \sum_{i\geq 0, i\neq k,l} \sqrt{\mathbb{E}[(g_i(t, Z))^2]} \geq 0,$$

where $1 \leq k, l \leq m$, $k \neq l$.

We write $\mathcal{A}_{1-4}$ for the space of all reinsurance strategies satisfying the above four conditions. Meng et al. [10] showed that within the class of reinsurance policies that satisfy $\mathcal{A}_{1-4}$, a combined proportional reinsurance treaty is an optimal form. Indeed, the condition (4) is purely technical and was used in [10] to prove the optimality result of reinsurance strategy. As illustrated in Example 2 in Section 2 of Meng et al. [10], this purely technical condition rules out some reinsurance strategies. From both theoretical and practical perspectives, an interesting question may be what is an optimal form of reinsurance strategies if this purely technical condition is relaxed. From the practical perspective, the relaxation of the purely technical condition would enhance the applicability and practicality of the theoretical results in [10]. From the theoretical point of view, the relaxation of the purely technical condition would strengthen the universality of the optimal form of reinsurance strategies as a combined proportional reinsurance treaty by considering a (possibly) considerably larger space of reinsurance strategies. We shall discuss this interesting problem in this note and show that a combined proportional reinsurance strategy is still an optimal form even when the condition (4) is dropped.

3. Optimal reinsurance form

This section presents the main result of this note. The key mathematical tool used to establish the optimality result is the Lagrangian method. Note that the Lagrangian method was used in [10, Section 4], to derive an explicit solution to the optimal dividend problem within the class of combined proportional reinsurance strategies. Indeed, a considerable amount of effort has been given to discussing optimal forms of reinsurance treaties in continuous-time insurance risk models. Some examples of these works are Meng and Zhang [11], Meng and Siu [12–14], Meng [8], and Meng et al. [15,9], among others. For interested audience, please see [10] and the relevant references therein for related discussions. Here we use $\mathcal{A}_{1-3}$ for denoting the space of all reinsurance strategies only satisfying the conditions (1)–(3) in Section 1. In this section, we shall show that in the wider class $\mathcal{A}_{1-3}$ of reinsurance strategies, a combined proportional reinsurance treaty is still an optimal form.

Firstly, for any fixed reals $z \geq 0$, $\gamma > 0$, we adopt the Lagrangian function method to construct the following function:

$$f(y_0, y_1, \ldots, y_m) = -\sum_{j=1}^{m} \theta_j y_j^2 - \gamma y_0^2 + \frac{2z}{\sum_{j=1}^{m} \frac{1}{\theta_j} + \frac{1}{\gamma}} \left(\sum_{j=0}^{m} y_j - z\right), \tag{3.1}$$

where $(y_0, y_1, \ldots, y_m) \in \Re^{m+1}$.

It can be seen that (3.1) can be rewritten as:

$$\begin{aligned} f(y_0, y_1, \ldots, y_m) = & -\sum_{j=1}^{m} \theta_j (y_j - \bar{y}_j(\gamma, z))^2 - \gamma(y_0 - \bar{y}_0(\gamma, z))^2 \\ & - \frac{2z^2}{\sum_{j=1}^{m} \frac{1}{\theta_j} + \frac{1}{\gamma}} + \sum_{j=1}^{m} \theta_j (\bar{y}_j(\gamma, z))^2 + \gamma(\bar{y}_0(\gamma, z))^2, \end{aligned} \tag{3.2}$$