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Convergence rate of regime-switching trees

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Abstract. Considering a general class of regime-switching geometric random walks and a broad class of piecewise twice differentiable payoff functions, we show that convergence of option prices occurs at a speed of order $\mathcal{O}\left(n^{-\beta}\right)$, where $\beta = 1/2$ when the payoff is discontinuous and $\beta = 1$ otherwise.

1. Introduction

The acclaimed Black-Scholes model is the *common language* of security derivatives, and option prices are quoted using this model. In spite of this unparalleled triumph, the Black-Scholes model suffers from well known shortcomings. One of them is that the risk-neutral rate r and the volatility σ should not be constant. The regime-switching model provides an enhancement of the Black-Scholes model which alleviates this problem. In this model, the market-related price-determining parameters r and σ of the Black-Scholes model are jointly determined by an externally driven market-related *regime*. While there can be several different forces acting on the price of an option, in this model one force (*regime*) is a dominating factor in setting the price, and the *state* of this regime is modelled to switch back and forth between finitely many modes. For instance, this could be the changes in preferences of the market agents [**24**] alternating between *bullish* and *bearish* expectations [**14, 16**] or, as in [**2**], alternating between *good* and *bad*. It can also be a business cycle [**4**] recurring from *expansion*, *transition*, and *contraction*. This price-driving force can also be determined by a hidden Markov process such as inside trading [**5**]. Numerous papers highlight that the regime-switching model is better than the Black-Scholes model in capturing the fat tails exhibited by empirical financial returns [**7, 6, 12, 8, 20, 3**]. In regime-switching models, asset prices evolve according to models determined by the *state* of some recurrently-switching *regimes* which are driven by unobserved factors resulting in stationary *regime-state* changes following each other independently.

In its simplest form, the regime has two states, 1 and 2, and the risk-free rate and volatility are fully determined by this state. For simplicity, this paper focuses on two-state regime-switching models. In an abstract form, a (two-state) regime-switching model $\Xi := \Xi\left(\alpha, \xi^1, \xi^2\right)$ is composed of three independent components: a stochastic model α_t governing the state of the regime, and two independent

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