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Rank constrained matrix best approximation problem with respect to (skew) Hermitian matrices

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ABSTRACT

In this literature, we study a rank constrained matrix approximation problem in the Frobenius norm:

$$\min_{r(X)=k} \left\| BXB^* - A \right\|_F^2,$$

where k is a nonnegative integer, A and X are (skew) Hermitian matrices. By using the singular value decomposition and the spectrum decomposition, we derive some conditions for the existence of (skew) Hermitian solutions, and establish general forms for the (skew) Hermitian solutions to this matrix approximation problem.

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1. Introduction

Let $\mathbb{C}^{m\times n}$ denote the set of all $m \times n$ matrices over the complex field $\mathbb{C}$, $\mathbb{C}_H^{m\times m}$ denote the set of all $m \times m$ Hermitian matrices, $\mathbb{C}_{SH}^{m\times m}$ denote the set of all $m \times m$ skew Hermitian matrices, $\mathbb{U}_n$ denote the set of all $n \times n$ unitary matrices, I_n denote the identity matrix of order n. For $A \in \mathbb{C}^{m\times n}$, its rank, conjugate transpose and Moore–Penrose inverse are denoted by $r(A)$, A^* and $A^\dagger$ respectively. For Hermitian matrix A, its positive and negative indexes of inertia are symbolled by $i_+(A)$ and $i_-(A)$ respectively. Furthermore, the symbol $\|A\|_F$ denotes the Frobenius norm of $A \in \mathbb{C}^{m\times n}$ defined as $\|A\|_F^2 = trace(AA^*)$. For a real number α, its absolute value is denoted by $|\alpha|$, the sign function is defined as

$$sign(\alpha) = \begin{cases} 1, & \alpha > 0, \\ 0, & \alpha = 0, \\ -1, & \alpha < 0. \end{cases}$$

In order to describe our results clearly, an important arrangement for real numbers is defined below.

Definition 1.1. Giving t nonzero real numbers n_i $(i = 1, \ldots, t)$, we call "$n_1, n_2, \ldots, n_t$" a model-order if the following conditions are satisfied,

(1) $|n_1| \geqslant |n_2| \cdots \geqslant |n_t| > 0$;
(2) When $|n_i| = |n_{i+1}|$ $(i = 1, 2, \ldots, t-1)$, if $sign(n_i) \neq sign(n_{i+1})$, then $n_i > 0$, $n_{i+1} < 0$.

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For example, the series "−4, 3, −3, −3" is a model-order, but "−4, −3, 3, −3" is not.

For a given Hermitian matrix $A \in \mathbb{C}_H^{m\times m}$, denote its nonzero eigenvalues by $\lambda_1, \ldots, \lambda_{r(A)}$ with the model-order. Giving an integer $k \in [0, r(A)]$, let $n_+(k, A)$ and $n_-(k, A)$ denote the numbers of positive eigenvalues and negative eigenvalues in the first k eigenvalues of A respectively. If for skew Hermitian matrix A with nonzero eigenvalues $i\lambda_1, \ldots, i\lambda_{r(A)}$, where $i^2 = -1$, $\lambda_1, \ldots, \lambda_{r(A)}$ are real numbers arranged in the same way, and $n_+(k, A)$ and $n_-(k, A)$ are defined similarly on these real numbers. Obviously, both $n_+(k, A)$ and $n_-(k, A)$ are increased, and $0 \leqslant n_+(k, A) \leqslant k$, $0 \leqslant n_-(k, A) \leqslant k$, $n_+(k, A) + n_-(k, A) = k$.

Recently, findings in the research of rank matrix approximation problems have been so widely applied to signal processing, control theory, numerical algebra, and so on [1–5]. For example, Wei and Wang [6] derived a rank-k Hermitian nonnegative definite least squares solution to the equation $BXB^H = A$ in the Frobenius norm and discussed the ranges of the rank k. Wang [7] considered the following rank constrained matrix approximation problem in the Frobenius norm. Sou and Rantzer [5], and Wei and Shen [8] studied minimum rank matrix approximation problems in the spectral norm and applied their findings to control theory. Especially, Wei and Shen [9] studied the minimum rank Hermitian (skew-Hermitian) solutions to the following problems in the spectral norm

$$r(X_1) = \min_{Y\in\mathfrak{S}_1} r(Y) \quad \text{subject to } \mathfrak{S}_1 = \left\{Y = Y^* \in \mathbb{C}^{m\times m} : \left\|A - BYB^*\right\|_2 = \min\right\}$$

and

$$r(X_2) = \min_{Y\in\mathfrak{S}_2} r(Y) \quad \text{subject to } \mathfrak{S}_2 = \left\{Y = -Y^* \in \mathbb{C}^{m\times m} : \left\|A - BYB^*\right\|_2 = \min\right\},$$

by applying the Hermitian-type (skew-Hermitian-type) generalized singular value decomposition. Friedland and Torokhti [10], Wang [11] considered the following rank constrained matrix approximation problem in the Frobenius norm

$$\min_{r(X)=k} \|AXB - C\|_F^2,$$

where A, B, C, and X are general matrices without any restrictions, this problem is related to the generalized Karhunen–Loeve transform, which is a well known signal processing technique for data compression and filtering (see [12–16] for more details).

On the other hand, problems of Hermitian-type (skew-Hermitian-type) linear Hermitian matrix functions and matrix equations have also been considered. Dai and Lancaster [17] and Liu and Tian [18] studied the Hermitian solution of the matrix equation $BXB^* = A$. Li et al. [19] studied the Hermitian least squares solution of $AXB = C$. Khatri and Mitra [20] gave a necessary and sufficient condition for the existence of Hermitian and nonnegative definite solutions to the linear matrix equation $AX = B$. Liu [21,22] considered Hermitian solutions of $AXA^* = B$ or $AX = B$ subject to $CXC^* \geq D$. Guo and Huang [23], Wang and Guo [24] presented representations for the minimal rank of $A - BXC$ with respect to Hermitian matrix X, and the minimal ranks of $A - BX$ and $A - BX - YD$ with respect to Hermitian matrix X and Y [25,24].

Motivated by the above work, in this paper, we focus our research interest on the following rank constrained matrix approximation problem

$$\min_{r(X)=k} \left\|BXB^* - A\right\|_F^2, \tag{1.1}$$

where $A \in \mathbb{C}_H^{m\times m}$ (or $A \in \mathbb{C}_{SH}^{m\times m}$) and $B \in \mathbb{C}^{m\times n}$ are known, $X \in \mathbb{C}_H^{n\times n}$ (or $X \in \mathbb{C}_{SH}^{n\times n}$) is variable. To our knowledge, this problem has not been investigated yet in the Frobenius norm case.

Let $B \in \mathbb{C}^{m\times n}$ with $r(B) = r_B$, and the singular value decomposition

$$B = U \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} V^*.$$

Correspondingly, partition matrices $A \in \mathbb{C}_H^{m\times m}$ and $X \in \mathbb{C}_H^{n\times n}$ as

$$A = U \begin{pmatrix} A_1 & A_2 \\ A_2^* & A_3 \end{pmatrix} U^* \quad \text{and} \quad X = V \begin{pmatrix} \Sigma^{-1}X_1\Sigma^{-1} & X_2 \\ X_2^* & X_3 \end{pmatrix} V^*, \tag{1.2}$$

where A_1, A_3, X_1 and X_3 are Hermitian matrices with proper sizes, and the spectrum decomposition of A_1 is

$$A_1 = U_1 \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{pmatrix} U_1^*, \quad U_1 \in \mathbb{U}_{r_B}, \tag{1.3}$$

where $\Sigma_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_t)$ with $t = r(A_1) = r(B^*AB)$, and $\lambda_1, \ldots, \lambda_t$ is a model-order.
Therefore,

$$BXB^* - A = U \begin{pmatrix} X_1 - A_1 & -A_2 \\ -A_2^* & -A_3 \end{pmatrix} U^*,$$

$$\left\|BXB^* - A\right\|_F^2 = \|A\|_F^2 - \left\|BB^\dagger ABB^\dagger\right\|_F^2 + \|X_1 - A_1\|_F^2. \tag{1.4}$$

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