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Radial basis functions method for valuing options: A multinomial tree approach

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ABSTRACT

From the view point of probability, this study presents a theoretical framework to show the convergence of the RBFs method for valuing options. It will be proved to be equivalent to a multinomial tree approach, in which the underlying variable can move from its initial value to an infinity of possible values of the next time step. Specially, the probability of a move in a short period time follows the normal distribution when using the Gaussian basis kernel, it is a precise simulation of the behavior of the underlying variable, which provides a more reasonable explanation of high-accuracy of the RBFs method. This helps open a new area of research in developing the expected numerical method for derivative securities (in which the underlying asset follows other stochastic process) by using corresponding radial basis kernel. The paper also illustrates the approach by using it to value stock options and its Greek letters.

1. Introduction

In the early 1970s, F. Black, M. Scholes, and R. Merton made a major breakthrough in the pricing of stock options, which involved the development of what has become known as the Black–Scholes model [1,2]. In a risk-neutral word, Black and Scholes presented an analytical formula for evaluating European options. Unfortunately, no exact analytic formula for the value of an American put option on a non-dividend-paying stock. As we know that the American option pricing can be treated as a free boundary problem [3]. Until recently, a lot of numerical procedures and analytic approximations for calculating American option values have been developed. For example, the binomial tree method (BTM) by Cox et al. [4]; the finite difference approach by Brennan and Schwartz [5]; the projected successful over-relaxation approach by Wilmott et al. [6]; the front-fixing finite difference method by Wu and Kwok [7]; the Monte Carlo simulation by Grant et al. [8]; the integral equation method by Huang et al. [9]; the adaptive mesh model by Figlewski et al. [10]; the penalty method by Forsyth, Khaliq and Nielsen et al. [11–13]; the mesh free method by Fasshauer et al. [14]; the iterative method by Salmi et al. [15], etc. A comparison of some numerical methods can be found in Broadie and Detemple's review paper [16].

Lately, a radial basis functions (RBFs) method for solving options pricing model is proposed by Hon and Mao [17]. This method involves two steps, first the approximation of spatial derivatives with RBFs interpolation converts Black–Scholes equation into a system of ordinary differential equations (ODEs), then, integrate the resulting system of ODEs in time. The method is a meshless computational algorithm which does not require the generation of a grid as in finite difference method

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or a mesh as in the finite element method. This makes the RBFs method particularly efficient in solving such kind of free boundary problem. Moreover, it can be used to calculate the Greek letters of the option directly, since the radial kernels are infinitely continuously differentiable. Besides, RBFs method offers a highly accurate approximation to the solution in numerical experiments.

There are two alternative ways of using the RBFs for the numerical solution of the time-dependent partial differential equations (PDEs). The first, as shown in [17], transforms the PDEs into a system of ODE of the unknown coefficients, then the solution can be valued by solving the unknown coefficients. The second transforms the PDEs into a system of ODE of the solution, and the solution can be directly obtained by using any time integration scheme. In this study, the second approach is taken. In this way, the RBFs method will be confirmed to be a multinomial tree, in which the stock price moves from its initial value to all values of the next time step. Specially, the probability of a move in a short period time follows the normal distribution when using the Gaussian basis kernel, it is a precise approximation to the process followed by the underlying variable. This offers a more reasonable explanation of high-accuracy of the method.

The layout of the paper is as follows. Section 2 describes the RBFs collocation method for the options pricing model. Section 3 shows the consistency of the RBFs method and the corresponding PDEs, illustrates its relation to tree approaches and discusses its probability distribution of a short time step. Section 4 applies the method to value options and its Greek letters. The last section is dedicated to a brief conclusion.

2. RBFs method for solving Black–Scholes equation

2.1. Black–Scholes model

As is common in the risk neutral world, the underlying stock price S is assumed to follow the lognormal diffusion process (which is also famous as geometric Brownian motion):

$$dS = rSdt + \sigma SdW$$

where dW is a Winner process and r and σ represent the risk-free interest rate and volatility, respectively. It is well known that the value at time t of the option on the stock price solves the following Black–Scholes PDE:

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 v}{\partial S^2} + rS\frac{\partial v}{\partial S} = rv \tag{2.1}$$

where $v(S, t)$ is the option value at time t and stock price S. The terminal condition is given by the maximum payoffs valuation

$$v(S, T) = \begin{cases} \max\{K - S, 0\}, & \textit{for put} \\ \max\{S - K, 0\}, & \textit{for call} \end{cases} \tag{2.2}$$

where T is terminal time and K is the strike price of the option. A simple transformation $S = \exp(x)$ changes Eq. (2.1) and the terminal boundary conditions (2.2) into

$$\frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2} + \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial u}{\partial x} = ru \tag{2.3}$$

and

$$u(x, T) = \begin{cases} \max\{K - \exp(x), 0\}, & \textit{for put} \\ \max\{\exp(x) - K, 0\}, & \textit{for call}. \end{cases} \tag{2.4}$$

2.2. RBFs interpolation

RBFs interpolation, one of meshfree approximation methods, is a very useful and convenient tool for approximation problems. It has been employed for solving PDEs intensively, for example [18–22] solve PDEs by means of the collocation with RBFs. The advantages of the approach are high-order accurate, flexible with respect to the geometry, computationally efficient, and easy to implement. The RBFs method performs well in many calculations including the numerical experiments that are reported by Franke [23]. For more information about the meshless method, we refer readers to the book [24] and the reference therein. An RBF depends only on the distance to a center point x_j and is of the form $\phi(\|x - x_j\|)$. The RBF may also have a parameter c, in which case $\phi(r)$ is replaced with $\phi(r, c)$. Some of the most popular RBFs are listed in Table 1. To be specific, given data $\{x_j, f(x_j)\}_{j=1}^{M}$, where x_j are some nodes in the domain of the problems and M is the number of nodes. The RBFs interpolant of a function f is defined

$$f(x) \sim f^*(x) = \sum \lambda_j \phi(\|x - x_j\|), \quad x \in \mathbb{R}^d \tag{2.5}$$