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Shape Preserving Rational Cubic Fractal Interpolation Function

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Abstract

A new type of $\mathcal{C}^1$ Fractal Interpolation Function (FIF) is developed using the Iterated Function System (IFS) which contains the rational spline. The numerator of this rational spline contains cubic polynomial and the denominator of the rational spline contains quadratic polynomial. We find uniform error bound between the original function which belongs to the class $\mathcal{C}^2$ and the FIF. We described suitable conditions on scaling factors and shape parameters such that it preserves the shape properties which inherited in the data.

Keywords: Iterated function system, Fractal interpolation function, Positivity, Monotonicity, Convexity.

1. Introduction

Suppose the data $\mathcal{D} = \{(x_i,\ y_i) \in I \times \mathbb{R} :\ i = 1, 2, \ldots, N\}$ is given, where $x_1 < x_2 < \cdots < x_N$ and $I = [x_1, x_N]$. Interpolation is the process of constructing a continuous function $\Phi : I \to \mathbb{R}$ such that $\Phi(x_i) = y_i$ for all $i = 1, 2, \ldots, N$. The classical interpolants (polynomial, spline etc.) are infinitely differentiable or piecewise infinitely differentiable. In many situations, data comes from numerical experiments are highly irregular. So classical interpolation methods becomes unsuitable to interpolate these data. To interpolate irregular data, Barnsley [1] introduced a new interpolation method called Fractal Interpolation using special type of iterated function system. In order to approximate differentiable functions Barnsley and Harrington [2] introduced differentiable fractal interpolation functions. With the help of Barnsley and Harrington results, various classical spline methods are generalized for instance [3–5].

In many situations, it is required that interpolant should reflect the geometric characteristics of the data set. Constructing interpolant with sufficiently smooth and preserving geometric characteristics of the data is called shape preserving interpolation. To preserve shape properties of the data, various spline interpolants are developed, for instance [6–10]. The uniqueness of spline interpolation becomes unsuitable for shape modification problem. Späth [11] introduced rational function with shape parameters to preserve geometric characteristic attached to data set. Also, various researchers [12–19] have constructed shape preserving rational splines using shape parameters. Using fractal interpolation functions, Chand and coworkers [20–24] have initiated study on shape preserving.

In this paper, a new $\mathcal{C}^1$ fractal interpolation function using rational IFS which contains three families of shape parameters is constructed in such a way that it preserves shape properties of the data. The proposed scheme has many outstanding features.

- The proposed method is a best tool to approximate a function that is continuous and its derivatives are irregular (see Section 5).
- When all the scaling factors are zero, fractal interpolation function that obtained from proposed method, reduces into a classical rational cubic spline (see Remark 2 and Section 5).
- The proposed method is equally applicable for the data with derivatives or data without derivatives.

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