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Asymptotical properties of social network dynamics on time scales

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ABSTRACT

In this paper we develop conditions for various types of stability in social networks governed by *Imitation of Success* principle. Considering so-called Prisoner's Dilemma as the base of node-to-node game in the network we obtain well-known Hopfield neural network model. Asymptotic behavior of the original model and dynamic Hopfield model has a certain correspondence. To obtain more general results, we consider Hopfield model dynamic system on time scales. Developed stability conditions combine main parameters of network structure such as network size and maximum relative nodes' degree with the main characteristics of time scale, nodes' inertia and resistance, rate of input–output response.

1. Introduction

A social network is the set of people or groups of people with some pattern of links or interconnection between them. Processes taking place on social networks often may be interpreted as information transition.

The aim of this paper is to consider asymptotic properties of collective opinion formation in social networks with general topology. Transition of opinion between linked nodes will be modeled by game-theoretical mechanism. Total payoff may be a key factor to choose one of the two alternative strategies, cooperation or defection in opinion propagation. Such type of dynamics is called *Imitation of Success*. An opposite (in some sense) kind of model is for example the *Voter Model*. Last named model assumes imitation of a behavior of uniformly random chosen neighbor node and games payoff has no effects on state updating of the particular node. In their paper Manshadi and Saberi [1] considered a model called *Weak Imitation of Success*. This updating rule is mixture of IS and VM rules: dependently of some parameter ε behavior of updating node may be close to one of the two types of dynamics.

The analysis of the total payoff function for so-called Prisoner's Dilemma leads us to well-known Hopfield neural network model Hopfield [2]. Asymptotic behavior of the direct node-to-node model and dynamic Hopfield model has a certain correspondence. To obtain more general results, we consider Hopfield model on time scale. This problem is discussed in detail in [3], but we develop more direct and precise conditions for stability of the social network behavior.

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Table 1
Classification of time scale's points.

t right-scattered	$t < \sigma(t)$
t right-dense	$t = \sigma(t)$
t left-scattered	$\rho(t) < t$
t left-dense	$\rho(t) = t$
t isolated	$\rho(t) < t < \sigma(t)$
t dense	$\rho(t) = t = \sigma(t)$

2. Preliminary results

We now present some basic information about time scales according to Bohner and Peterson [4]. A time scale is defined as a nonempty closed subset of the set of real numbers and denoted by $\mathbb{T}$. The properties of the time scale are determined by the following three functions:

(i) the forward-jump operator: $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$;
(ii) the backward-jump operator: $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$ (in this case, we set $\inf \varnothing = \sup \mathbb{T}$ and $\sup \varnothing = \inf \mathbb{T}$);
(iii) the granularity function $\mu(t) = \sigma(t) - t$.

The behavior of the forward- and backward-jump operators at a given point of the time scale specifies the type of this point. The corresponding classification of points of the time scale is presented in Table 1.

We define a set $\mathbb{T}^\kappa$ in the following way:

$$\mathbb{T}^\kappa = \begin{cases} \mathbb{T} \setminus \{M\}, & \text{if } \exists \text{ right scattered point } M \in \mathbb{T}: \ M = \sup \mathbb{T}, \sup \mathbb{T} < \infty \\ \mathbb{T}, & \text{otherwise.} \end{cases}$$

In what follows, we set $[a, b] = \{t \in \mathbb{T} : a \leqslant t \leqslant b\}$.

Definition 1. Let $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^\kappa$. The number $f^\Delta(t)$ is called Δ-derivative of function f at the point t, if $\forall \varepsilon > 0$ there exists a neighborhood U of the point t (i.e., $U = (t - \delta, t + \delta) \cap \mathbb{T}, \delta < 0$) such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \leqslant \varepsilon|\sigma(t) - s| \quad \forall s \in U.$$

Definition 2. If $f^\Delta(t)$ exists $\forall t \in \mathbb{T}^\kappa$, then $f : \mathbb{T} \to \mathbb{R}$ is called Δ-differentiable on $\mathbb{T}^\kappa$. The function $f^\Delta(t) : \mathbb{T}^\kappa \to \mathbb{R}$ is called the delta-derivative of a function f on $\mathbb{T}^\kappa$.

If f is differentiable with respect to t then $f(\sigma(t)) = f(t) + \mu(t)f^\Delta(t)$.

Definition 3. The function $f : \mathbb{T} \to \mathbb{R}$ is called regular if it has finite right limits at all right-dense points of the time scale $\mathbb{T}$ and finite left limits at all points left-dense points of $\mathbb{T}$.

Definition 4. The function $f : \mathbb{T} \to \mathbb{R}$ is called *rd*-continuous if it is continuous at the right-dense points and has finite left limits at the left-dense points. The set of these functions is denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}; \mathbb{R})$.

The indefinite integral on the time scale takes the form

$$\int f(t)\Delta t = F(t) + C,$$

where C is integration constant and $F(t)$ is the preprimitive for $f(t)$. If the relation $F^\Delta(t) = f(t)$ where $f : \mathbb{T} \to \mathbb{R}$ is an *rd*-continuous function, is true for all $t \in \mathbb{T}^\kappa$ then $F(t)$ is called the primitive of the function $f(t)$. If $t_0 \in \mathbb{T}$ then $F(t) = \int_{t_0}^t f(s)\Delta s$ for all t. For all $r, s \in \mathbb{T}$ the definite Δ-integral is defined as follows:

$$\int_r^s f(t)\Delta t = F(s) - F(r).$$

Definition 5. For any regular function $f(t)$ there exists a function F differentiable in the domain D and such that the equality $F^\Delta(t) = f(t)$ holds for all $t \in D$. This function is defined ambiguously. It is called the preprimitive of $f(t)$.

Definition 6. A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive (positive regressive) if

$$1 + \mu(t)p(t) \neq 0, \quad (1 + \mu(t)p(t) > 0), \qquad t \in \mathbb{T}^\kappa.$$

The set of regressive (positive regressive) and *rd*-continuous functions is denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T})$ $(\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}))$.