ARTICLE IN PRESS

Journal of Computational and Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Two algorithms to construct a consistent first order theory of equilibrium figures of close binary systems

José A. López Ortí*, Manuel Forner Gumbau, Miguel Barreda Rochera

Departamento de Matemáticas, Universidad Jaume I, Av Sos Baynat s/n 12071 Castellón, Spain

ARTICLE INFO

Article history:
Received 22 June 2016
Received in revised form 12 December 2016

MSC:
70E50
70F15
70M20
70G10

Keywords:
Computational algebra
Perturbation theory
Close binary systems
Potential theory
Spherical harmonics

ABSTRACT

One of the main problems in celestial mechanics is the study of the shape adopted by extended deformable celestial bodies in its equilibrium configuration. In this paper, a new point of view about classical theories on equilibrium figures in close binary systems is offered.

Classical methods are based on the evaluation of the self-gravitational, centrifugal and tidal potentials. The most common technique used by classical methods shows convergence problems. To solve this problem up to first order in amplitudes two algorithms has been developed, the first one based on the Laplace method to develop the inverse of the distance and the second one based on the asymptotic properties of the numerical quadrature formulas.

1. Introduction

The main aim of this work is to develop a consistent theory to determine the self-gravitational potential at an arbitrary point of each component of a detached close binary system in a state of slow rotation, when its equilibrium configuration has been reached. This means that the binary system is observed from a coordinate system rotating in association with the studied component, once the hydrostatic equilibrium is reached. The configuration of hydrostatic equilibrium is equivalent to a state of rigid rotation, which corresponds to the minimum potential [1–3]. In accordance, among others [1,4,3], this state implies the identification of equipotential, isobaric and isopycnic surfaces.

The mechanisms by which a close binary system reaches its equilibrium state have been studied among others by [5,6]. This process implies circularizing the orbit and synchronizing the rotation and translation movements. Let M_1 and M_2 be two masses and R the distance between the center of mass of its components. The equilibrium state implies Kepler's third law

$$\omega^2 = G\frac{M_1 + M_2}{R^3}. \tag{1}$$

The state of the close binary system can be modeled by the equations:

$$\vec{\nabla} P = \rho \vec{\nabla} \Psi \tag{2}$$

$$\triangle \Psi = -4\pi G\rho + 2\omega^2 \tag{3}$$

* Corresponding author.
E-mail address: lopez@mat.uji.es (J.A. López Ortí).

http://dx.doi.org/10.1016/j.cam.2016.12.015
0377-0427/

where P is the pressure, ρ is the density, Ψ is the total potential, $\triangle$ is the *Laplace* operator, G is the constant of universal gravitation and $\overrightarrow{\omega}$ is the angular velocity of the system. To integrate these equations in a general case of mass distribution, a state equation, $P = P(\rho)$, relating pressure with density is required.

To assess the full potential is necessary to calculate the self-gravitational potential Ω, the centrifugal potential V_c and the tidal potential V_t.

The physical effects produced by the disturbing potentials (centrifugal and tidal) are deformations of the equipotential surfaces of the components with respect to the spherical symmetry that stars at rest would possess.

To study the potential of a generic point of the close binary system, it is convenient to define the coordinate system *OXYZ*. On it, *O* is the center of mass of the primary component, *OX* is the axis rotating in the direction of the secondary component, *OZ* is an axis in the direction of the angular velocity of the system and *OY* is defined in a way that *OXYZ* may form a direct trihedral. The value of V_c and V_t in this reference system is described by Finlay [1], Kopal [2,4] and López [7,8].

The solution of the problem can be addressed on the basis of the well-known equations determining the structure of a self-gravitatory fluid, initially isolated, which leads to a solution with spherical symmetry.

From this initial solution, the general problem includes centrifugal forces due to rotation and tidal forces generated by the secondary component. This problem can be analyzed from the solution of an unperturbed problem using the perturbation theory, successive approximations of the density, pressure and temperature data, and the definition of the shape of the component as the lower surface containing it.

An alternative treatment, due to Clairaut [3], is to consider that density, pressure and temperature of each component are only function of the total potential, so that the complete solution of the problem is equivalent to determine the equipotential surfaces. An advantage of this procedure is that pressure does not appear explicitly in the basic equations which determine the equipotential surfaces. The equations depend on the density, remaining invariant with the form it takes.

In the Clairaut method, the figure of each component is defined by the lower equipotential surface containing the component. The study of the close binary systems has been addressed using the Clairaut method by several authors [4,7], extending the technique used by Finlay and Kopal, among others, to the problem of equilibrium figures for rotating isolated bodies. Kopal [4] studies the amplitudes of the deformations with respect to the sphere by adding the terms of rotation, tide and the rotation-tide interaction. On the other hand, Lopez [7] studies the problem globally, up to the first order, using as a basis for the developments of equipotential surfaces the spherical functions.

Unfortunately, the method used by these authors [4,7] offers some problems regarding the convergence of the used developments and its solution depends on the desideratum of Laplace, which has not been yet demonstrated. In case of rotating figures, these problems have been studied by López [8]. The study shows inconsistencies in Kopal's approach. To overcome these disadvantages, López [8] introduces two new algorithms which do not require compliance with the desideratum of Laplace. López shows that, up to first order, the inner and outer terms of the self-gravitational potential given by the Kopal theory are wrong. However, the full potential up to the first order coincides with the one obtained by Kopal. This demonstrates that the results of Kopal theory are consistent because they do not depend on Laplace desideratum.

This paper focuses on extending the results obtained for rotating deformable bodies showing rotational symmetry to the most general case of the close binary systems. In this case we show that the terms of the self-gravitational potential calculated using classical methods are not correct. Despite this, it is shown that the total potential, up to first order in the small amount ω^2, coincides with that obtained by classical methods. In the case we are treating, spherical functions and their products are involved, which makes it far more complex than the problem considering only rotation, because in this case only the Legendre polynomials and their products must be considered.

In this section, the general backgrounds of the problem are explained. On them, the inconsistency of the classical theory due to its reliance on the unproven hypothesis known as the desideratum of Laplace is shown. It is also shown the need to build new methods to solve the first order theory for the close binary systems which are independent of the Laplace desideratum.

Section 2 describes further the classical method proposed by Finlay and Kopal, based on the method of Clairaut as well as its extension for close binary systems. In this section, difficulties offered by the classical theory due to the dependence of this theory with the unproven desideratum of Laplace are also shown.

In Section 3, a new algorithm based on an extension of the method used by López [8] for close binary systems is shown. This method is based on a modified Laplace method for calculating the inverse of the distance between two bodies moving in elliptical orbits. This method obviates the need of depending on Laplace desideratum to evaluate, up to first order, the self-gravitational potential in a point of a component of a close binary system.

Section 4 introduces a second algorithm that extends the technique developed by López [8], to close binary systems. This method, consisting on a generalization of the Wavre method [3], permits to obtain the rigorous calculation of the different terms of the self-gravitational potential of each of the components. These terms do not coincide with those obtained by the classical theory but, nevertheless, it is also shown that, up to first order in ω^2, the self-gravitational potential coincides with the one obtained by classical theory.

In Section 5, the main conclusions of this work and its comparison with the results obtained by the classical theory are discussed.

Please cite this article in press as: J.A. López Ortí, et al., Two algorithms to construct a consistent first order theory of equilibrium figures of close binary systems, Journal of Computational and Applied Mathematics (2017), http://dx.doi.org/10.1016/j.cam.2016.12.015