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Fast numerical valuation of options with jump under Merton's model☆

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ABSTRACT

In this paper, we consider discontinuous Galerkin (DG) finite element together with finite difference (FD) scheme for solving Merton's jump–diffusion model, which is given by a partial integro-differential equations (PIDEs). Spatial differential operators are discretized using FD on a uniform grid, and time stepping is performed using the DG finite element method. The treatment of the integral term associated with jumps in models is more challenging. The discretization of this integral term will lead to full matrices for the non-locality of the integral operator. To fast solve this model, multigrid method is used for solving such linear algebraical system. Numerical comparison of multigrid method and GMRES method shows that multigrid method is superior to and more effective than GMRES method in solving the dense algebraic systems resulting from the FD approximations of the PIDEs.

1. Introduction

One of the modern financial theory's biggest successes in terms of both approach and applicability has been the Black–Scholes option pricing model developed by Fisher Black and Myron Scholes in 1973 [1] and previously by Robert Merton [2]. The celebrated Black–Scholes model is based on assumption that the price of the underlying asset behaves like a geometric Brownian motion with a drift and a constant volatility which cannot explain the market prices of options with various strike prices and maturities. To explain these behavior, a number of alternative models have appeared in the financial literatures, for example, nonlinear models [3–6] and jump-diffusive models [7–9], which are given by a partial integro-differential equation (PIDE). However, these models are more difficult to handle numerically in contrast to the celebrated Black–Scholes model. If we use an implicit method for the time discretization, we should solve a nonlinear system for nonlinear models, and a non-symmetric dense system for jump-diffusive models. We have proposed two classes of splitting

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methods for solving nonlinear option pricing problems [10,11]. In this study, it was aimed to provide an efficient method for numerically solving jump–diffusion models, especially, the Merton's jump–diffusion model, which will lead to a dense system due to the non-locality of the integral operator.

There has been much research on pricing options under jump models using finite difference (FD) methods, which is the most common way to discretize the differential operators in the option pricing context (see, for example, [12,13]). In 1997, Zhang [14] proposed an implicit–explicit time integral method that treats the integral term explicitly and the differential terms implicitly for American options with Merton's model. This method is a first-order accurate method and has a stability restriction for the time stepsize. Tavella and Randall in [12] considered using a fully implicit time stepping method to price European options and a stationary iterative method to solve the resulting dense problems with a full matrix. Andersen and Andreasen proposed an unconditionally stable, second-order accurate alternating direction implicit (ADI) type operator splitting method with two fractional steps for European options in [15]. For American options, in [16] d'Halluin, Forsyth, and Labahn used a penalty method and the Crank–Nicolson method with adaptive time steps, and an approximate semismooth Newton method for the resulting nonlinear nonsmooth problems. Briani, La Chioma, and Natalini in [17] proposed a fully explicit time stepping method for European options which leads to a more severe stability restriction. In 2005, on a nonuniform spatial grid, d'Halluin, Forsyth, and Vetzal [18] developed a method in which to use the fast Fourier transform (FFT) for evaluating the integral term on a uniform grid they perform interpolations back and forth on nonuniform and uniform grids for European options under Merton's and Kou's model; Almendral and Oosterlee [19] used the BDF2 method for time discretization, FFT for the integrations, and the iterative method proposed in [12] for linear systems; Cont and Voltchkova [20] proposed an implicit–explicit time integral method that treats the integral term explicitly and the differential terms implicitly for pricing European options in Exponential Lévy models. Toivanen [21] developed a numerical method for pricing European and American options under Kou's jump–diffusion model by using FD on nonuniform grid for discretizing spatial differential operators, the implicit Rannacher scheme for the time stepping, and, a stationary iteration for the resulting dense linear systems. Recently, Salmi and Toivanen in [22] proposed an iterative method for pricing American options under jump–diffusion models.

One of the greatest challenges for numerically solving jump–diffusion models is how to reduce the computation costs. The above research suggests that there are three main time discretization approaches, the implicit–explicit scheme that treats the integral term explicitly and the differential terms implicitly which will lead to a stability restriction for the time stepsize [14,20], the fully explicit scheme leading to a more severe stability restriction [17], and, the fully implicit scheme which will produce dense systems with full matrices [12,16,19,21,22]. Reducing the computation costs for jump–diffusion models is harder than doing it for the original Black–Scholes model when a fully implicit scheme is used. Some methods have already been designed to overcome this difficulty, such as ADI [15], FFT [18,19], and, iterative methods [12,16,19,21,22]. It is known that multigrid method is optimal iterative procedure, which has been widely used for PDEs (see e.g., [23,24]). In this paper, we introduce a multigrid method on each time level to solve linear algebraic systems resulting from the FD approximations of the PIDEs.

The rest of the paper is organized as follows: for the sake of completeness, in Section 2, we discuss the Merton's model and the corresponding option pricing problems. In Section 3, we consider the FD spatial discretization and present some basic theoretical properties on the matrices resulting from this discretization. To exploit the time analyticity of the solution for $t > 0$ and to cope with the loss of this analyticity at $t = 0$ [25], the time discretization is performed using the DG time stepping scheme. This will be discussed in Section 4. In Section 5, we present our multigrid methods. Finally, in Section 6, numerical comparison of multigrid method and GMRES method, which has been used to solve the resulting linear problems in [25,26], suggests that multigrid method is superior to and more effective than GMRES in solving the dense algebraic systems resulting from the FD approximations of the PIDEs.

2. Merton's model and option pricing problems

Let $V(t, S)$ be the value of a European contract that depends on the time t and underlying asset price S, which is given by a process of the form

$$\frac{dS}{S} = \nu dt + \sigma dz + (\eta - 1)dq, \tag{2.1}$$

where ν is the drift rate, σ is the volatility of the Brownian part of the process, $\eta - 1$ is an impulse function giving a jump from S to $S\eta$, and dq is a Poisson process and assumed to be independent of the Wiener process dz. Here, $dq = 0$ with probability $1 - \lambda dt$, $dq = 1$ with probability λdt, where λ is the Poisson arrival intensity.

Under the above assumptions it is well known (Merton, 1976, [7]) that $V(t, S)$ satisfies a final value problem defined by the following PIDE:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda\kappa)S\frac{\partial V}{\partial S} - (r + \lambda)V + \lambda I(V(t, S)) = 0, \tag{2.2}$$

where r is the risk-free interest rate, κ denotes the average relative jump size, $\mathbb{E}(\eta - 1)$, and $I(V(t, S))$ denotes the integral

$$I(V(t, S)) = \int_0^\infty V(t, S\eta)\rho(\eta)d\eta.$$