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Stabilization of switched linear systems by using projections

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ABSTRACT

This paper proposes a new approach to stabilize switched linear systems. In this method, the projections are employed to establish the stabilization of a switched systems class. Hence, we suppose that the subsystems given by a switched linear system can be projected to the same subspace. Under these conditions, we prove that this switched linear system is stabilizable if and only if a low-order switched linear system is also stabilizable. In order to complete this study, we present a counter example that proves that it is not always possible to use the projections. Moreover, the main result in the paper is applied to solve the stabilization of a third-order switched systems class and the static feedback stabilization of a switched linear systems class. Finally, a numerical example is included in order to illustrate the new method obtained to stabilize a third-order switched systems class.

1. Introduction

In recent years, the study of switched systems has received more and more attention; see, e.g., [1–3], among many others. This is because switched systems have strong applications in Control Theory. One of the main issues in the study of switched systems is stability.

The problems of stability of switched systems include two aspects: one is how to make switched systems stable or stabilized under arbitrary switching law, where each switching subsystem is required to be stable (see [1,4–6]). One of the main results is obtained in [7], where it is shown that the switched linear system is exponentially stable under arbitrary switching laws if the Lie algebra generated by the subsystems is solvable. In particular, this implies that the matrices can be written in a triangular form, then the system is stable under arbitrary switching. However, for the nonlinear case, as demonstrated in [8], the triangular structure alone is not sufficient for stability.

The second aspect of the stability is how to design a switching law under which switched systems are stable or stabilized, where each switched subsystem is unstable or unstabilizable. We can find some conditions that guarantee the existence of a *stabilizing* switching law [9–12]. For second-order switched systems, methods for stabilizing switched systems have been presented [13–15]. In [13,15] there are necessary and sufficient conditions that solve the problem of stabilization of switched systems consisting of unstable second-order linear subsystems. Furthermore, the papers [14,16] employ these ideas to study the nonlinear case and solve the linear case for two subsystems with different equilibrium types.

In this paper, we present a new result about the stabilization of a switched systems class. This new result establishes that the stabilization of these switched systems is equivalent to the stabilization of a low-order switched system. Hence, in this case, the problem of stabilization is reduced to study switched systems in a lower dimension.

As an application of this result, the results in [13,15] are used to study the stabilization of higher order switched systems by projecting the trajectory of the system to some 2-dimensional subspaces. We find these subspaces and establish conditions for their existence.

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The remainder of this paper is organized as follows. In Section 2 the problem is formulated and a preliminary result is enunciated. Section 3 deals with the main result aforementioned. In Section 4 we present a counter example of the extension of the main result to another switched linear systems class. In Sections 5 and 6 two applications of the result in Section 3 are presented. Finally, in Section 7 we provide the conclusions.

2. Preliminaries

In this paper, given a family of real $n \times n$ matrices $\{A_p : p \in \mathcal{P}\}$, where the index set $\mathcal{P}$ is an arbitrary compact set, switched linear systems of the following form are studied

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{1}$$

where $x \in \mathbb{R}^n$ and $\sigma : [0, \infty) \longrightarrow \mathcal{P}$ is a piecewise constant function called *switching law* indicating the active subsystem at each instant of time.

Given a linear subspace W of $\mathbb{R}^n$, we will say that W is an invariant subspace of the family of matrices $\{A_p : p \in \mathcal{P}\}$ if $A_pW \subset W$ for all $p \in \mathcal{P}$. The following proposition will be used later.

Proposition 1. *If W is a linear subspace of $\mathbb{R}^n$ of dimension $m \leq n$ and $\{v_1, v_2, \ldots, v_m\}$ is a basis of W, there exist $n - m$ vectors, $e_{m+1}, e_{m+2}, \ldots, e_n$ such that*

$$\{v_1, \ldots, v_m, e_{m+1}, \ldots, e_n\}$$

is a basis in $\mathbb{R}^n$.

Hence, if W is an invariant subspace of the previous family of matrices, applying the previous proposition and defining the matrix

$$P = [v_1, \ldots, v_m, e_{m+1}, \ldots, e_n],$$

it is obtained that

$$P^{-1}A_pP = \begin{bmatrix} B_p & C_p \\ \overline{0} & D_p \end{bmatrix},$$

where B_p is a $m \times m$ matrix, C_p is a $m \times (n-m)$ matrix, $\overline{0}$ is the $(n-m) \times m$ matrix identically zero, and D_p is a $(n-m) \times (n-m)$ matrix. Then, we have obtained two new families of square matrices, $\{B_p : p \in \mathcal{P}\}$ and $\{D_p : p \in \mathcal{P}\}$.

For notational simplicity, we introduce the linear function $p_{n-m} : \mathbb{R}^n \longrightarrow \mathbb{R}^{n-m}$ that assigns to each $x \in \mathbb{R}^n$ the last $n-m$ coordinates of x in the basis given by $\{v_1, \ldots, e_n\}$, i.e., $p_{n-m}(x_1, x_2, \ldots, x_n) = (x_{m+1}, \ldots, x_n)$ where $(x_1, x_2, \ldots, x_n)$ are the coordinates of x in this basis.

Before proceeding to the next section, we introduce some definitions about stability of a system. Consider the system given by

$$\dot{x} = f(x, t), \qquad x(t_0) = x_0, \tag{2}$$

where $x \in \mathbb{R}^n$ and $t \geq 0$. If we assume that f is *piecewise* continuous with respect to t, i.e., there are only finitely many discontinuity points in any compact set and $f(0, t) = 0$ for any $t \geq 0$, we can introduce the following definition (see more details in [17]).

Definition 1. The equilibrium point $x = 0$ is a locally exponentially stable equilibrium point of (2) if there exist $k, \alpha > 0$ such that

$$\|x(t)\| < ke^{-\alpha(t-t_0)}\|x(t_0)\|$$

for all x_0 in a neighborhood of the origin and $t \geq t_0 \geq 0$.

Global exponential stability is defined by requiring the previous inequality to hold for all $x_0 \in \mathbb{R}^n$. We always consider systems for which the origin is an equilibrium point and, with some abuse, we say that a system is exponentially stable, meaning that the origin is an exponentially stable equilibrium point of the system.

3. Main result

Under the previous notation, we present the main result in this work.

Theorem 1. *Let $\{A_p : p \in \mathcal{P}\}$ be a family of $n \times n$ matrices where $\mathcal{P}$ is compact. Suppose that there exists W an invariant subspace of this family of matrices. Thus, under the previous notation, if B_p is a stable matrix, for all $p \in \mathcal{P}$, and $\dot{y}(t) = B_{\sigma(t)}y$ is exponentially stable under a switching law σ, we have that:*

The switched system $\dot{x} = A_\sigma x$ is exponentially stable under the switching law σ if and only if the switched system $\dot{z} = D_\sigma z$ is also exponentially stable under the same switching law σ.