Accepted Manuscript

Numerically stable improved Chebyshev-Halley type schemes for matrix sign function

Alicia Cordero, F. Soleymani, Juan R. Torregrosa, M. Zaka Ullah

PII: S0377-0427(16)30514-3
DOI: http://dx.doi.org/10.1016/j.cam.2016.10.025
Reference: CAM 10863

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 15 June 2016
Revised date: 25 October 2016

Please cite this article as: A. Cordero, F. Soleymani, J.R. Torregrosa, M.Z. Ullah, Numerically stable improved Chebyshev-Halley type schemes for matrix sign function, *Journal of Computational and Applied Mathematics* (2016), http://dx.doi.org/10.1016/j.cam.2016.10.025

Numerically stable improved Chebyshev-Halley type schemes for matrix sign function ☆

Alicia Cordero[a], F. Soleymani[b], Juan R. Torregrosa[a,*], M. Zaka Ullah[c]

[a]*Instituto de Matemática Multidisciplinar, Universitat Politècnica de València Camino de Vera s/n, 46022 València, Spain*
[b]*Department of Mathematics, Institute for Advanced Studies in Basic Sciences, P.O. Box 45195-1159, Zanjan, Iran*
[c]*Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia*

Abstract

A general family of iterative methods including a free parameter is derived and proved to be convergent for computing matrix sign function under some restrictions on the parameter. Several special cases including global convergence behavior are dealt with. It is analytically shown that they are asymptotically stable. A variety of numerical experiments for matrices with different sizes is considered to show the effectiveness of the proposed members of the family.

Keywords: Matrix sign function; stability; iterative methods; Chebyshev-Halley family; eigenvalues.

1. Motivation

It is known that the function of sign in the scalar case is defined for any $z \in \mathbb{C}$ not on the imaginary axis by

$$\text{sign}(z) = \begin{cases} 1, & \text{Re}(z) > 0, \\ -1, & \text{Re}(z) < 0. \end{cases}$$

An extension of this function for the matrix case was given firstly by Roberts in [18], who introduced the matrix sign function as a tool for model reduction and for solving algebraic Riccati equations.

The problem of computing a function of a matrix, named by $f(A)$, is of growing significance, though as yet numerical methods are developed for this purpose. In between, the matrix sign function is undoubtedly of crystal clear importance in the theory and application of matrix functions (e.g., one may refer to [3, 6, 20]). The matrix sign function has basic theoretical and algorithmic relations with the matrix square root, the polar decomposition and with the matrix pth roots (see for more [10, chapter 5]). For example, a large class of iterations for the matrix square root can be obtained from corresponding iterations for the matrix sign function, and due to this discussing and designing new iterative schemes for finding matrix sign function is requisite.

The matrix sign function is a valuable tool for the numerical solution of Sylvester and Lyapunov matrix equations [1]. A generalization of the Newton iteration for the matrix sign function to the solution of the generalized algebraic Bernoulli equations was presented in [2]. This matrix function is also used in [17] as a simple and direct method to derive some fundamental results in the theory of surface waves in anisotropic materials. For other applications of matrix sign function, we refer the reader to [16, 23]. Due to the applicability of the matrix sign function, stable iterative schemes have become some viable choices for approximating this function.

Here we suppose that matrix $A \in \mathbb{C}^{n\times n}$ has no eigenvalues on the imaginary axis. To define this matrix function formally, let $A = PJP^{-1}$ be the Jordan canonical form arranged so that $J = \text{diag}(J_1, J_2)$, where the eigenvalues of

☆This research was supported by Ministerio de Economía y Competitividad MTM2014-52016-C2-2-P and by Generalitat Valenciana PROMETEO/2016/089.
*Corresponding author
Email addresses: acordero@mat.upv.es (Alicia Cordero), fazlollah.soleymani@gmail.com. (F. Soleymani), jrtorre@mat.upv.es (Juan R. Torregrosa), mzhussain@kau.edu.sa (M. Zaka Ullah)