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Revised trigonometrically fitted two-step hybrid methods with equation dependent coefficients for highly oscillatory problems

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ABSTRACT

For the numerical integration of highly oscillatory problems, revised trigonometrically fitted two-step hybrid methods (RTFTSH) with equation dependent coefficients are considered. The local truncation errors, stability and phase properties of the new method are analyzed. A feature of the new type of the methods is that the errors in the internal stages are assumed to contribute to the accuracy of the update. A new revised method RTFTSH4 of algebraic order four and phase-lag order four is derived. Numerical experiments are reported to show that the new method RTFTSH4 is much more efficient and robust than the standard fourth order method STFTSH4.

1. Introduction

In this paper, we are interested in the numerical integration of the initial value problem of the second order differential equation of the form

$$\begin{cases} y''(x) = f(x, y), \\ y(x_0) = y_0, \qquad y'(x_0) = y'_0, \end{cases} \tag{1}$$

whose solution exhibits a pronounced oscillatory character. This type of problems arise frequently in a variety of applied fields such as mechanics, astrophysics, quantum chemistry, electronics and so on. Traditionally the problem (1) is integrated with a general purpose Runge–Kutta(–Nyström) method or a linear multi-step method. In the past two decades, trigonometrical/exponential fitting techniques have been developed to construct numerical methods with frequency-dependent coefficients which have been proved to be very effective for solving the problem (1) with oscillatory solutions (see [1–19]). In standard derivations of EF Runge–Kutta(–Nyström), the contribution of the errors in the internal stages to the error of the final stage is completely neglected, i.e., $Y_i = y(x + c_i h)$. D'Ambrosio et al. revisited the EF-based Runge–Kutta(–Nyström) method with two stages by considering the error in the internal stages in [20,21]. Ixaru [22] constructed fourth order explicit A-stable Runge–Kutta methods with three stages by evaluating the errors in each internal stage individually and using them in an ad-hoc way in the computation of the coefficients of the external stage. Motivated by the previous work, in this paper, we will investigate trigonometrically fitted two-step hybrid method with equation dependent coefficients. In the construction of the new method the contribution of the errors of the internal stages to the final stage will be taken into account.

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2. Construction of TFTSH methods

For the numerical integration of (1), an s-stage explicit two-step hybrid method has the scheme

$$\begin{cases} Y_1 = y_{n-1}, \\ Y_2 = y_n, \\ Y_i = (1+c_i)y_n - c_i y_{n-1} + h^2 \sum_{j=1}^{i-1} a_{ij} f(x_n + c_j h, Y_j), \quad i = 3, \ldots, s, \\ y_{n+1} = 2y_n - y_{n-1} + h^2 \sum_{i=1}^{s} b_i f(x_n + c_i h, Y_i), \end{cases} \tag{2}$$

which can be expressed compactly by the Butcher tableau

$$\begin{array}{c|c} c & A \\ \hline & b^T \end{array} = \begin{array}{c|ccc} c_1 & a_{11} & \cdots & a_{1s} \\ \vdots & \vdots & \ddots & \vdots \\ c_s & a_{s1} & \cdots & a_{ss} \\ \hline & b_1 & \cdots & b_s \end{array}$$

The order conditions of the two step hybrid methods were presented in Coleman [23].

In particular, we consider a three-stage explicit two-step hybrid method of the form

$$\begin{array}{c|ccc} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c_3 & a_{31} & a_{32} & 0 \\ \hline & b_1 & b_2 & b_3 \end{array} \tag{3}$$

The choice of $\{c_3; a_{31}, a_{32}; b_1, b_2, b_3\} = \{1; 0, 1; \frac{1}{12}, \frac{5}{6}, \frac{1}{12}\}$ recovers a fourth order with three stages which was given by Franco [24]. Next we shall first introduce a standard trigonometrically fitted two-step hybrid method and then the revised method by considering the contribution of the error in the internal stages to the error in the final stage with this table, respectively.

To the internal stage Y_3 of the method (3), we introduce the operator for the internal stage

$$L_3[h, a]y(x)|_{x=x_n} = y(x_n + c_3 h) - (1+c_3)y(x_n) + c_3 y(x_{n-1}) - h^2(a_{31}y''(x_{n-1}) + a_{32}y''(x_n)). \tag{4}$$

We require the operator (4) to vanish for functions in the reference set

$$\{1,\ x, e^{i\omega x},\ e^{-i\omega x}\} \tag{5}$$

leading to

$$\begin{aligned} &L_3[h, a]1|_{x=x_n} = 0, \\ &L_3[h, a]x|_{x=x_n} = 0, \\ &L_3[h, a]e^{i\omega x}|_{x=x_n} = e^{i\omega x}(e^{ic_3\theta} - (1+c_3) + c_3 e^{-ic_3\theta} + \theta^2(a_{31}e^{-i\theta} + a_{32})), \\ &L_3[h, a]e^{-i\omega x}|_{x=x_n} = e^{-i\omega x}(e^{-ic_3\theta} - (1+c_3) + c_3 e^{ic_3\theta} + \theta^2(a_{31}e^{i\theta} + a_{32})). \end{aligned}$$

Solving the last two equations we obtain

$$a_{31} = \frac{-c_3 + \csc(\theta)\sin(c_3\theta)}{\theta^2}, \qquad a_{32} = \frac{1 + c_3 - \cos(c_3\theta) - \cot(\theta)\sin(c_3\theta)}{\theta^2}, \quad \theta = \omega h. \tag{6}$$

2.1. Standard TFTSH methods

To the update of the method (3) we assign an operator

$$L^S[h, b]y(x)|_{x=x_n} = y(x_n + h) - 2y(x_n) + y(x_n - h) - h^2(b_1 y''(x_n - h) + b_2 y''(x_n) + b_3 y''(x_n + c_3 h)), \tag{7}$$

where the superscript S indicates a standard method. Standard trigonometrically fitted two-step hybrid (STFTSH) methods do not take into account the effect of internal stages. Under the local assumption that $Y_1 = y(x_n - h)$, $Y_2 = y(x_n)$ and $Y_3 = y(x_n + c_3 h)$, we require (7) to be exact for the linear combination of the functions in the set

$$\{1,\ x,\ x^2,\ e^{i\omega x},\ e^{-i\omega x}\} \tag{8}$$

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