Accepted Manuscript

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PII: S0377-0427(16)30418-6
DOI: http://dx.doi.org/10.1016/j.cam.2016.08.049
Reference: CAM 10788

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 17 June 2016

Please cite this article as: A. Cordero, C. Jordán, J.R. Torregrosa, A dynamical comparison between iterative methods with memory: Are the derivatives good for the memory?, *Journal of Computational and Applied Mathematics* (2016), http://dx.doi.org/10.1016/j.cam.2016.08.049

A dynamical comparison between iterative methods with memory: are the derivatives good for the memory? ☆

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Abstract

The role of the derivatives at the iterative expression of methods with memory for solving nonlinear equations is analyzed in this manuscript. To get this aim, a known class of methods without memory is transformed into different families involving or not derivatives with an only accelerating parameter, then they are defined as discrete dynamical systems and the stability of the fixed points of their rational operators on quadratic polynomials are studied by means of real multidimensional dynamical tools, showing in all cases similar results. Finally, a different approach holding the derivatives, and by using different accelerating parameters, in the iterative methods involved present the most stable results, showing that the role of the appropriated accelerating factors is the most relevant fact in the design of this kind of iterative methods.

Keywords: Nonlinear equations, iterative method with memory, stability, bifurcation, basin of attraction, dynamical plane.

1. Introduction

The design of iterative methods for solving nonlinear equations or systems, $f(x) = 0$, is a challenging task that has proved to be bountiful in the last decades. From the Kung-Traub's conjecture [10], many authors have devoted their efforts in designing efficient optimal methods of increasing order of convergence. Kung and Traub also worked on iterative methods with memory, but it has been very recently when this kind of schemes have been re-discovered and many authors have dedicated their efforts in constructing new schemes with better convergence properties than their known partners. In this terms, the early works of Traub [17], Neta [12] and recent ones by Petković et al. [14, 15], Lotfi et al. [4, 11], Wang et al. [18], among others, give a close idea of the general interest on these methods. However, all of these researches focused their works on the design of the methods trying to improve their numerical aspects (convergence, number of functional evaluations, accelerators, etc.); only some recent works [1] approach this problem by means of the stability analysis, trying to find the anomalies and advantages of methods with memory. To get this aim, previous results in real discrete dynamics have been used (see, for example, [5, 7, 9, 16]).

Our goal in this paper is to carry out a dynamical study of some methods with memory with and without derivatives with a common without-memory partner. As the fixed point iteration functions have more than one variable, some auxiliary functions are introduced to facilitate the calculations. So, specific dynamical concepts are adapted to achieve the appropriate numerical sense.

On the other hand, we also analyze the local convergence of each method with memory under study. For it, we use the following result, that can be found in [13].

Theorem 1. *Let ψ be an iterative method with memory that generates a sequence $\{x_k\}$ of approximations to the root α, and let this sequence converges to α. If there exist a nonzero constant η and nonnegative numbers t_i, $i = 0, 1, \ldots, m$, such that the inequality*

$$|e_{k+1}| \leq \eta \prod_{i=0}^{m} |e_{k-i}|^{t_i}$$

☆This research was partially supported by Ministerio de Economía y Competitividad MTM2014-52016-C02-2-P.

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