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Attribute and size reduction mechanisms in multi-adjoint concept lattices

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ABSTRACT

Attribute reduction and size reduction in concept lattices are key research topics in Formal Concept Analysis (FCA). This paper combines both strategies in the multi-adjoint concept lattice framework in order to simplify the information provided by the original context. Specifically, we present three procedures which merge the attribute reduction and the size reduction by means of an irreducible α-cut concept lattice, analyzing the obtained properties.

1. Introduction

Formal Concept Analysis (FCA) is a tool in charge of extracting pieces of information from databases which contain a set of attributes A and a set of objects B together with a relation between them $R \subseteq A \times B$. These pieces of information are called concepts and they are hierarchized in order to obtain concept lattices. This mathematical tool is closely related to soft set theory and rough set theory, which have also fulfilled applied, for example, in decision making [1,2].

In order to deal with uncertainties and imprecise data, fuzzy sets [3,4] were considered in FCA due to their numerous properties and applications [5–7]. To the best of our knowledge, the first extension was given in [8], although it did not advance far beyond the basic definitions, probably because residuated implications were not used. Subsequently, in [9,10], the authors independently used complete residuated lattices as structures for the truth degrees.

It is well known that the complexity regarding the construction of concept lattices decreases if the number of attributes is previously reduced, which is widely studied in the classical case [11–14]. However, the process of attribute reduction is more difficult in the fuzzy case [15,16]. Although one of the main profits of the procedure, presented in [15], is that the original concept lattice is conserved, this fact can become a drawback if the obtained concept lattice is very big and illegible. Therefore, it is also necessary to study mechanisms for decreasing the size of concept lattices.

Different strategies can be found in the literarture in order to reduce the size of concept lattices, such as the use of hedges in the concept-forming operators [17–21] and the methodology provided by granular computing [22]. Another interesting mechanism dedicated to size reduction is based on the meet-irreducible elements of the lattice and on the use of a cut value given by the user [23–25]. This method provides a sublattice of the original concept lattice, which is called meet-irreducible α-cut concept lattice. Consequently, the most representative knowledge is preserved since the original concepts are not modified.

In this paper, we merge the philosophies and the features of both reduction mechanisms in order to introduce new procedures. First of all, we will carry out the attribute reduction to obtain a concept lattice isomorphic to the original one

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and then the size reduction strategy. After that, we will perform the combination of both reduction methodologies in the reverse order. Finally, a third procedure will be given which will consider the properties of the two previous ones. We will study the properties and the influence of these combinations on the final classification of the set of attributes and on the size of the reduced concept lattices.

The structure of this paper is the following: Section 2 presents a brief summary with basic definitions and results. Sections 3 and 4 remind the attribute reduction based on the characterizations of the absolutely necessary, relatively necessary and absolutely unnecessary attributes in terms of the meet-irreducible elements of the concept lattice, and the meet-irreducible α-cut concept lattices, respectively. Section 5 introduces the new mechanisms combining the attribute reduction with the meet-irreducible α-cut concept lattices, several properties and remarks. Lastly, the paper finishes with the conclusions and prospects for future work.

2. Preliminary notions

In order to make the paper self-contained, a summary of some basic definitions and results is presented.

Definition 1. Given a lattice $(L, \preceq)$, such that $\wedge, \vee$ are the meet and the join operators, and an element $x \in L$ verifying

1. If L has a top element $\top$, then $x \neq \top$.
2. If $x = y \wedge z$, then $x = y$ or $x = z$, for all $y, z \in L$.

We call x *meet-irreducible* ($\wedge$*-irreducible*) *element* of L. Condition (2) is equivalent to

2′. If $x < y$ and $x < z$, then $x < y \wedge z$, for all $y, z \in L$.

Hence, if x is $\wedge$-irreducible, then it cannot be represented as the infimum of strictly greater elements. A *join-irreducible* ($\vee$*-irreducible*) *element* of L is defined dually.

In this paper, we consider the general fuzzy setting of multi-adjoint concept lattices in which the basic operators to make the calculus are adjoint triples. Next, we recall this fuzzy concept lattice, which was introduced in [26].

Definition 2. Let $(P_1, \leq_1)$, $(P_2, \leq_2)$, $(P_3, \leq_3)$ be posets and $\&\colon P_1 \times P_2 \to P_3$, $\swarrow\colon P_3 \times P_2 \to P_1$, $\nwarrow\colon P_3 \times P_1 \to P_2$ be mappings, then $(\&, \swarrow, \nwarrow)$ is an *adjoint triple* with respect to P_1, P_2, P_3 if:

$$x \leq_1 z \swarrow y \quad \text{iff} \quad x \& y \leq_3 z \quad \text{iff} \quad y \leq_2 z \nwarrow x \tag{1}$$

where $x \in P_1$, $y \in P_2$ and $z \in P_3$. Condition (1) is called *adjoint property*.

In the concept lattice environment, we need to consider that $(P_1, \leq_1)$ and $(P_2, \leq_2)$ are complete lattices.

Definition 3. A *multi-adjoint frame* $\mathcal{L}$ is a tuple $(L_1, L_2, P, \&_1, \dots, \&_n)$ where $(L_1, \preceq_1)$ and $(L_2, \preceq_2)$ are complete lattices, $(P, \leq)$ is a poset and $(\&_i, \swarrow^i, \nwarrow_i)$ is an adjoint triple with respect to L_1, L_2, P, for all $i \in \{1, \dots, n\}$.

The notion of multi-adjoint context considers a relational database with an extra mapping.

Definition 4. Let $(L_1, L_2, P, \&_1, \dots, \&_n)$ be a multi-adjoint frame, a *context* is a tuple (A, B, R, σ) such that A and B are non-empty sets (usually interpreted as attributes and objects, respectively), R is a P-fuzzy relation $R\colon A \times B \to P$ and $\sigma\colon A \times B \to \{1, \dots, n\}$ is a mapping which associates any element in $A \times B$ with some particular adjoint triple in the frame.

From now on, we will denote as L_2^B and L_1^A the set of mappings $g\colon B \to L_2$ and $f\colon A \to L_1$, respectively. On these sets a pointwise partial order can be considered from the partial orders in $(L_1, \preceq_1)$ and $(L_2, \preceq_2)$ providing L_2^B and L_1^A the structure of complete lattice. Abusing notation, $(L_2^B, \preceq_2)$ and $(L_1^A, \preceq_1)$ are complete lattices whose orderings are given by the following relation: $g_1 \preceq_2 g_2$ if and only if $g_1(b) \preceq_2 g_2(b)$ ($f_1 \preceq_1 f_2$ if and only if $f_1(a) \preceq_1 f_2(a)$, respectively), for all $g_1, g_2 \in L_2^B$ and $b \in B$ (for all $f_1, f_2 \in L_1^A$ and $a \in A$, respectively).

Once we have fixed a multi-adjoint frame and a context for that frame, the concept-forming operators ${}^{\uparrow_\sigma}\colon L_2^B \longrightarrow L_1^A$ and ${}^{\downarrow^\sigma}\colon L_1^A \longrightarrow L_2^B$ can be defined, for all $g \in L_2^B$, $f \in L_1^A$ and $a \in A$, $b \in B$, as

$$g^{\uparrow_\sigma}(a) = \inf\{R(a, b) \swarrow^{\sigma(a,b)} g(b) \mid b \in B\} \tag{2}$$

$$f^{\downarrow^\sigma}(b) = \inf\{R(a, b) \nwarrow_{\sigma(a,b)} f(a) \mid a \in A\}. \tag{3}$$

These operators form a Galois connection [26]. Hereon, we will write $^\uparrow$ and $^\downarrow$ instead of ${}^{\uparrow_\sigma}$ and ${}^{\downarrow^\sigma}$, respectively.

With respect to the *multi-adjoint concepts*, they are defined as pairs $\langle g, f\rangle$ satisfying that $g \in L_2^B$, $f \in L_1^A$ and that $g^\uparrow = f$ and $f^\downarrow = g$; being $(^\uparrow, {}^\downarrow)$ the Galois connection defined above. The fuzzy subset of objects g (resp. fuzzy subset of attributes f) is called *extension* (resp. *intension*) of the concept and the extension set (resp. intension set) is denoted as $\mathrm{Ext}(\mathcal{M})$ (resp. $\mathrm{Int}(\mathcal{M})$).

Given $g \in L_2^B$ (resp. $f \in L_1^A$), *the generated concept from g (resp. f)* is $\langle g^{\uparrow\downarrow}, g^\uparrow\rangle$ (resp. $\langle f^\downarrow, f^{\downarrow\uparrow}\rangle$) and so, the extension of the least concept containing to g is $g^{\uparrow\downarrow}$.

The next definition introduces the notion of concept lattice in this framework.

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