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A computational method for solving a class of two dimensional Volterra integral equations

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Abstract. The use of the Geometric Series theorem, together with Schauder bases in certain Banach spaces, allows us to design a numerical algorithm in order to solve an important type of two-dimensional linear integral equations. A study of the convergence and error is presented here. All the calculations can be easily implemented and the efficiency of this method will be shown with numerical results.

Key words: Two dimensional integral equations, Schauder bases, Banach spaces, Geometric Series theorem, numerical methods.

2010 Mathematics Subject Classification: AMS 45A05, 45L05, 45N05, 65R20.

1 Introduction

Two–dimensional Volterra integral equations provide an important tool for modeling many problems in mathematics, physics and engineering. These equations appear, for example, in electromagnetic theory, in the quantum effects of electromagnetic fields in the blackbody whose interior is filled by a Kerr nonlinear crystal, in the description of the three-dimensional structure of water around globular solutes, and in the study of a travelling wave solution for a mathematical model describing the population change influenced by a uniformly changing environment (see [9], [10], [14] and [18]).

This paper is concerned with the following two-dimensional integral equations of Volterra:

$$f(s,t) = g(s,t) + \int_{\gamma}^{t}\int_{\alpha}^{s} K(s,t,x,y)f(x,y)\,dxdy \qquad (s,t) \in \Omega \tag{1}$$

where $\Omega = [\alpha, \alpha+\beta] \times [\gamma, \gamma+\delta]$, $f \in C(\Omega)$ is the solution to be calculated, and g and K are given real–valued continuous functions defined, respectively, on Ω and $W = \{(s,t,x,y) \in \mathbb{R}^4 : \alpha \le x \le s \le \alpha+\beta,\ \gamma \le y \le t \le \gamma+\delta\}$.

On many occasions it is not possible to find an exact solution of (1), and there are many different numerical methods for solving such equations – for example, the Chebyshev polynomials through collocation scheme ([1]), the two-dimensional orthogonal triangular functions ([2]), the differential transformation ([12], [13] and [19]), and the moving least square ([15]) can be applied to approximate a solution for integral equations.