Accepted Manuscript

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PII: S0377-0427(16)30096-6
DOI: http://dx.doi.org/10.1016/j.cam.2016.02.040
Reference: CAM 10537

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 24 July 2015
Revised date: 1 February 2016

Please cite this article as: M. García-Olivo, J.M. Gutiérrez, .A. Magreñán, A first overview on the real dynamics of Chebyshev's method, *Journal of Computational and Applied Mathematics* (2016), http://dx.doi.org/10.1016/j.cam.2016.02.040

A first overview on the real dynamics of Chebyshev's method

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Abstract

In this paper we explore some properties of the well known root-finding Chebyshev's method applied to polynomials defined on the real field. In particular we are interested in showing the existence of extraneous fixed points, that is fixed points of the iteration map that are not root of the considered polynomial. The existence of such extraneous fixed points is an specific property in the dynamical study of Chebyshev's method that does not happen in other known iterative methods as Newton's or Halley's methods. In addition, in this work we consider other dynamical aspects of the method as, for instance, the Feigenbaum bifurcation diagrams or the parameter plane.

Keywords: Chebyshev's method, nonlinear equations, iterative methods, real dynamics.
2010 MSC: Primary 65P99.

1. Introduction

In this paper we refer as Chebyshev's method the following iterative scheme for solving nonlinear equations $f(x) = 0$, with $f : \mathbb{R} \to \mathbb{R}$: given an initial approximation x_0 to the sought root, a sequence $x_{n+1} = C_f(x_n)$ it is constructed, where

$$C_f(x) = x - \left(1 + \frac{1}{2}L_f(x)\right)\frac{f(x)}{f'(x)}, \tag{1}$$

and

$$L_f(x) = \frac{f(x)f''(x)}{f'(x)^2}. \tag{2}$$

Note that the method can be seen as a real discrete dynamical system and not only as a root-finding method. With this point of view we are interested

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[1] The research is supported by the grant MTM2014-52016-C2-1-P of Ministerio de Economía y Competitivad, Spain.
[2] The research is supported by the research project Ref. MTM2014-52016-C2-1-P by the Spanish Ministry of Science and Innovation and by the Universidad Internacional de La Rioja (UNIR, http://www.unir.net), under the Plan Propio de Investigación, Desarrollo e Innovación [2013–2015]. Research group: Matemática aplicada al mundo real (MAMUR).