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V.G. Pimenov, A.S. Hendy, R.H. De Staelen

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On a class of non-linear delay distributed order fractional diffusion equations

V. G. Pimenov[a,b], A. S. Hendy[b], R. H. De Staelen[c]

[a] *Institute of Mathematics and Mechanics, Ural Branch of the RAS, 620000, Yekaterinburg, 16 st. S.Kovalevskoy, Russia*
[b] *Department of Computational Mathematics, Institute of Mathematics and Computer Science, Ural Federal University, ul. Mira. 19, Yekaterinburg 620002, Russia*
[c] *Department of Mathematical Analysis, research group of Numerical Analysis and Mathematical Modeling (NaM^2), Ghent University, Gent 9000, Belgium*

Abstract

In this paper, we consider a numerical scheme for a class of non-linear time delay fractional diffusion equation with distributed order in time. This study covers the unique solvability, convergence and stability of the resulted numerical solution by means of the discrete energy method. The derivation of a linearized difference scheme with convergence order $O(\tau + (\Delta\alpha)^4 + h^4)$ in L_∞-norm is the main purpose of this study. Numerical experiments are carried out to support the obtained theoretical results.

Keywords: Distributed order fractional partial differential equations, Difference scheme, Discrete energy method, Delay partial differential equations, Convergence, Stability.

1. Introduction

In the past few decades, high and rapid growing attention related with partial differential equations which contain fractional derivatives and integrals occurred. The ability of the models which contain non-integer orders comparing with integers order models in describing some certain phenomena is more accurate. The need of many scientific areas for the use of fractional partial differential equations (FPDEs) to describe their processes has been widely recognized. Nowadays, the interest of scientists with FPDEs in fields of finance [31], engineering [21], viscoelasticity [6], control systems [23], diffusion procedures [7] and many other scientific areas has no limit. Many anomalous diffusion processes which existed in some physical and biological areas can be modeled by the time fractional reaction diffusion wave equation [33, 36]. Recently, distributed order fractional differential equations can model perfectly some different problems in mathematical physics and engineering [9, 19]. As one of the realistic models of these equations, the authors in [1] transfer the multi-term fractional derivative viscoelastic model to a derivative model of distributed order and checked its effect on several systems such as the fractional distributed order oscillator and the distributed

Email addresses: vladimir.pimenov@urfu.ru (V. G. Pimenov), ahmed.hendy@fsc.bu.edu.eg (A. S. Hendy), rob.destaelen@ugent.be (R. H. De Staelen)