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A sixth-order iterative method for approximating the polar decomposition of an arbitrary matrix ☆

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Abstract

A new iterative method for computing the polar decomposition of any rectangular complex matrix is presented and analyzed. The study of the convergence shows that this method has order of convergence six. Some numerical tests confirm the theoretical results and allow us to compare the proposed iterative scheme with other known ones.

Keywords: Polar decomposition, singular value decomposition, matrix iteration, unitary factor, Hermitian matrix, iterative method
AMS Subject Classification: 65F25, 65F30.

1. Introduction

The *polar decomposition* is a generalization to complex matrices of the trigonometric representation of a complex number. Specifically, let A be a complex matrix of size $m \times n$, $m \geq n$ (in other case, we work with the transpose matrix). Then there exist a matrix $U \in \mathbb{C}^{m\times n}$, with orthonormal columns and a Hermitian positive semi-definite $H \in \mathbb{C}^{n\times n}$ such that

$$A = UH, \qquad U^*U = I_n, \tag{1}$$

where U^* denotes the conjugate transpose of U and I_n is the identity matrix of size $n \times n$. The Hermitian factor H is always unique and can be expressed as $H = (A^*A)^{1/2}$. If matrix A has full rank, then H is positive definite and the unitary factor U is uniquely determined.

Let us observe that, once the unitary factor U is calculated, the other factor is obtained in a simple way, $H = U^*A$. So, our goal in this work is to obtain factor U.

The polar decomposition is well known and can be found in many textbooks, for example, [1] and [2]. An early reference is [3]. This decomposition has many applications in several fields. In [4] the author describes different applications of the polar decomposition to factor analysis, aerospace computations and optimization. For example, the optimization method called Conjugate Gradient, for the minimization of $F(x)$, $F : \mathbb{R}^n \to \mathbb{R}$, is more stable when the Hessian matrix is replaced by the Hermitian factor of its polar decomposition. On the other hand, the square root of a positive definite matrix A is the Hermitian factor of the polar decomposition of L^T, where $A = LL^T$ is the Choleski decomposition of A. In addition, polar decomposition has many advantages in front of other decompositions in the context of computer graphics.

It is well known that the unitary factor U possesses a best approximation property and in [4] the author describes, under some conditions, good approximation properties of the Hermitian factor H. Other interesting

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