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Strang Splitting method to Benjamin Bona Mahony Type Equations: Analysis and Application

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Abstract

We provide an error analysis of operator splitting for equations of the type $u_t - \partial_x^2 u_t = Au + \frac{1}{2}\partial_x(u^2)$, where A is an unbounded linear differential operator such that the equation is well-posed. Two particular examples are generalized Benjamin-Bona-Mahony and KdV-BBM equations. A second order error bound of the Strang splitting method in time is proved under suitable regularity assumptions on the exact solution. Finally, the orders of convergence are checked by two numerical experiments and the errors are presented.

Keywords. Operator splitting method; Nonlinear PDE; KdV-BBM equation; Error bound

1 Introduction

In this paper, we study initial value problems of the type

$$u_t = (1-\partial_x^2)^{-1}P(\partial_x)u + \frac{1}{2}(1-\partial_x^2)^{-1}\partial_x(u^2), \;\; u(t_0) = u_0, \tag{1}$$

where $x \in \mathbb{R}$, $0 \le t \le T$, and P is a polynomial of degree $r \ge 2$ satisfying $ReP(i\xi) \le 0,$ for all $\xi \in \mathbb{R}$. This is a class of dispersive nonlinear wave equations including various important cases, e.g., $P(\xi) = \xi^2$ the generalized BBM equation [2, 3, 4] and $P(\xi) = \xi^3$ the KdV-BBM equation [5, 6].

A wide range of numerical methods have been studied to compute approximate solutions to dispersive wave equations of KdV-BBM type: the finite volume method (see [6]), finite element method (see [7]), and spectral method (see [8, 9]).

We use an operator splitting method whose basic idea is based on splitting a complex problem into simpler sub-problems, each of which is solved by an efficient method, (see [10, 11, 12]). We start by separating Eq. (1) into two sub-equations with an unbounded linear and a bounded nonlinear operator, respectively, i.e.,

$$Au = (1-\partial_x^2)^{-1}P(\partial_x)u, \quad B(u) = \frac{1}{2}(1-\partial_x^2)^{-1}\partial_x(u^2). \tag{2}$$

In the present paper, we use Strang splitting, i.e.

$$u_{n+1} = \Psi^{\Delta t}(u_n) = \Phi_A^{\Delta t/2} \circ \Phi_B^{\Delta t} \circ \Phi_A^{\Delta t/2}(u_n), \;\; n = 0, 1, 2, \ldots. \tag{3}$$

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