

## Accepted Manuscript

Strang splitting method to Benjamin Bona Mahony type equations:  
Analysis and application

Nurcan Gücüyen

PII: S0377-0427(15)00560-9

DOI: <http://dx.doi.org/10.1016/j.cam.2015.11.015>

Reference: CAM 10364

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 22 July 2015

Revised date: 13 October 2015



Please cite this article as: N. Gücüyen, Strang splitting method to Benjamin Bona Mahony type equations: Analysis and application, *Journal of Computational and Applied Mathematics* (2015), <http://dx.doi.org/10.1016/j.cam.2015.11.015>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Strang Splitting method to Benjamin Bona Mahony Type Equations: Analysis and Application

Nurcan Gücüyenlen \*

Gediz University, Faculty of Engineering and Architecture,  
Civil Engineering, 35665 Seyrek, Menemen, İzmir, Turkey

October 13, 2015

## Abstract

We provide an error analysis of operator splitting for equations of the type  $u_t - \partial_x^2 u_t = Au + \frac{1}{2}\partial_x(u^2)$ , where  $A$  is an unbounded linear differential operator such that the equation is well-posed. Two particular examples are generalized Benjamin-Bona-Mahony and KdV-BBM equations. A second order error bound of the Strang splitting method in time is proved under suitable regularity assumptions on the exact solution. Finally, the orders of convergence are checked by two numerical experiments and the errors are presented.

**Keywords.** Operator splitting method; Nonlinear PDE; KdV-BBM equation; Error bound

## 1 Introduction

In this paper, we study initial value problems of the type

$$u_t = (1 - \partial_x^2)^{-1}P(\partial_x)u + \frac{1}{2}(1 - \partial_x^2)^{-1}\partial_x(u^2), \quad u(t_0) = u_0, \quad (1)$$

where  $x \in \mathbb{R}$ ,  $0 \leq t \leq T$ , and  $P$  is a polynomial of degree  $r \geq 2$  satisfying  $ReP(i\xi) \leq 0$ , for all  $\xi \in \mathbb{R}$ . This is a class of dispersive nonlinear wave equations including various important cases, e.g.,  $P(\xi) = \xi^2$  the generalized BBM equation [2, 3, 4] and  $P(\xi) = \xi^3$  the KdV-BBM equation [5, 6].

A wide range of numerical methods have been studied to compute approximate solutions to dispersive wave equations of KdV-BBM type: the finite volume method (see [6]), finite element method (see [7]), and spectral method (see [8, 9]).

We use an operator splitting method whose basic idea is based on splitting a complex problem into simpler sub-problems, each of which is solved by an efficient method, (see [10, 11, 12]). We start by separating Eq. (1) into two sub-equations with an unbounded linear and a bounded nonlinear operator, respectively, i.e.,

$$Au = (1 - \partial_x^2)^{-1}P(\partial_x)u, \quad B(u) = \frac{1}{2}(1 - \partial_x^2)^{-1}\partial_x(u^2). \quad (2)$$

In the present paper, we use Strang splitting, i.e.

$$u_{n+1} = \Psi^{\Delta t}(u_n) = \Phi_A^{\Delta t/2} \circ \Phi_B^{\Delta t} \circ \Phi_A^{\Delta t/2}(u_n), \quad n = 0, 1, 2, \dots \quad (3)$$

---

\*email: nurcan.gucuyenen@gediz.edu.tr

Download English Version:

<https://daneshyari.com/en/article/5776458>

Download Persian Version:

<https://daneshyari.com/article/5776458>

[Daneshyari.com](https://daneshyari.com)