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Symplectic integrators for the matrix Hill equation

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Abstract

We consider the numerical integration of the matrix Hill equation. Parametric resonances can appear and this property is of great interest in many different physical applications. Usually, Hill's equations originate from a Hamiltonian function and the fundamental matrix solution is a symplectic matrix. This is a very important property to be preserved by the numerical integrators. In this work we present new sixth-and eighth-order symplectic exponential integrators that are tailored to Hill's equation. The methods are based on an efficient symplectic approximation to the exponential of high dimensional coupled autonomous harmonic oscillators and yield accurate results for oscillatory problems at a low computational cost. The proposed methods can also be used for solving general second order linear differential equations where their performance will depend on how the methods are finally adapted to each particular problem or the qualitative properties one is interested to preserve. Several numerical examples illustrate the performance of the new methods.

Keywords: matrix Hill equation; Mathieu equation; parametric resonance; symplectic integrators; Magnus expansion
2000 MSC: 65L07; 65L05 ; 65Z05

1. Introduction

The study of the potential of a charged particle moving in the electric field of a quadrupole, without considering the effects of the induced magnetic field, leads to the equations of motion

$$\frac{d^2}{dt^2}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-\frac{2e}{md}\left(E_0+E_1\cos(t)\right) & 0 & 0\\ 0 & \frac{2e}{md}\left(E_0+E_1\cos(t)\right) & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix},$$

where e is the charge of the particle, m is the mass and d is the minimum distance from the electrodes to the origin and the particle is traveling in the x_3

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