Accepted Manuscript

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PII: S0377-0427(16)30467-8
DOI: http://dx.doi.org/10.1016/j.cam.2016.09.040
Reference: CAM 10828

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 8 December 2015
Revised date: 26 August 2016

Please cite this article as: B. Düring, J. Miles, High-order ADI scheme for option pricing in stochastic volatility models, *Journal of Computational and Applied Mathematics* (2016), http://dx.doi.org/10.1016/j.cam.2016.09.040

High-order ADI scheme for option pricing in stochastic volatility models

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Abstract

We propose a new high-order alternating direction implicit (ADI) finite difference scheme for the solution of initial-boundary value problems of convection-diffusion type with mixed derivatives and non-constant coefficients, as they arise from stochastic volatility models in option pricing. Our approach combines different high-order spatial discretisations with Hundsdorfer and Verwer's ADI time-stepping method, to obtain an efficient method which is fourth-order accurate in space and second-order accurate in time. Numerical experiments for the European put option pricing problem using Heston's stochastic volatility model confirm the high-order convergence.

Keywords: Option pricing, stochastic volatility models, mixed derivatives, high-order ADI scheme
2000 MSC: 65M06, 91B28

1. Introduction

In financial option pricing, stochastic volatility models as the Heston model [20] have become one of the standard approaches. Unlike the classical Black & Scholes model [3] the volatility (or standard deviation) of the option's underlying asset is not assumed to be constant, but is modelled as a second, correlated stochastic diffusion process. This additional source of randomness allows to model option prices more accurately and to fit higher moments of the asset return distribution. Using Ito's lemma and standard arbitrage arguments, partial differential equations of convection-diffusion type with mixed second-order derivatives are derived for pricing options.

For some stochastic volatility models and under additional restrictions, closed-form solutions can be obtained by Fourier methods (e.g. [20], [15]). Another approach is to derive approximate analytic expressions, see e.g. [2] and the literature cited therein. In general, however, —even in the Heston model [20] when the parameters in it are non constant— the partial differential equations arising from stochastic volatility models have to be solved numerically. Moreover, many (so-called American) options feature an additional early exercise right. Then one has to solve a free boundary problem which consists of the partial differential equation and an early exercise constraint for the option price. Also for this problem one typically has to resort to numerical approximations.

In the mathematical literature, there are many papers on numerical methods for option pricing, mostly addressing the one-dimensional case of a single risk factor and using standard, second order

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