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Sample selection based on sensitivity analysis in parameterized model order reduction

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ABSTRACT

Modeling of scientific or engineering applications often yields high-dimensional dynamical systems due to techniques of computer-aided-design, for example. Thus a model order reduction is required to decrease the dimensionality and to enable an efficient numerical simulation. In addition, methods of parameterized model order reduction (pMOR) are often used to preserve the physical or geometric parameters as independent variables in the reduced order models. We consider linear dynamical systems in the form of ordinary differential equations. In the domain of the parameters, often samples are chosen to construct a reduced order model. For each sample point a common technique for model order reduction can be applied to compute a local basis. Moment matching or balanced truncation are feasible, for example. A global basis for pMOR can be constructed from the local bases by a singular value decomposition. We investigate approaches for an appropriate selection of a finite set of samples. The transfer function of the dynamical system is examined in the frequency domain, and our focus is on moment matching techniques using the Arnoldi procedure. We use a sensitivity analysis of the transfer function with respect to the parameters as a tool to select sample points. Simulation results are shown for two examples.

1. Introduction

In science and engineering, mathematical models are generated automatically, which causes the drawback that dynamical systems often exhibit a huge dimension. For an efficient numerical simulation, a model order reduction (MOR) is required to reduce the complexity of the problem. Hence MOR represents an important field in scientific computing, see [1,2], for example. In most of the cases, a lower dimensional dynamical system is derived, whose output should approximate the quantities of interest sufficiently accurately.

The dynamical systems include parameters like physical parameters, geometrical parameters or others. In several applications, a parameterized model order reduction (pMOR) is desired, where the parameters are preserved in the reduction such that a single reduced order model can be evaluated directly for all relevant parameter values, see [3]. Firstly, optimization problems require to evaluate the same model many times. Secondly, uncertainty quantification can be done by an examination of the system for a large number of samples from uncertain parameters.

In pMOR for linear dynamical systems, typically local bases are determined for a finite set of parameter samples. Now two different strategies exist. Firstly, the local bases are used to construct a global basis, where often a singular value

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decomposition is applied to the collection of all local bases. This global basis directly yields a reduced order model for an arbitrary parameter value. This technique was employed for pMOR in [4,5], for example. Secondly, approaches from nonlinear MOR can be generalized to the pMOR setting. In the trajectory piecewise linear technique, see [6,7], even though a global basis is constructed, a weighted sum of terms from several linearization points is derived. In pMOR, a weighted sum of several local bases for the different parameter samples can be arranged. This approach was investigated already in [8].

We consider pMOR for linear dynamical systems in this work. The first strategy above is applied to construct a single reduced order model for all parameter values. We investigate approaches for choosing the parameter samples where local bases are computed. This is done using global sensitivity analysis with the total effect sensitivity indices which can be found in [9,10]. This sensitivity analysis was applied to the transfer function of a linear dynamical system with random parameters in [11], where an MOR was derived inside the random space and thus pMOR was not considered. We now apply this concept to identify the parameters in pMOR which have the main impact on the quantities of interest in the dynamical system. Based on this information, the parameter points for the construction of the local bases are selected. An advantage is that this selection depends on information from the problem at hand, where the aim is to obtain an efficient pMOR. A substantial part of the paper consists of the presentation of numerical experiments applying the sensitivity analysis. We show results for two test examples, namely the heat transfer in a microthruster unit, see [12], and the two-dimensional heat equation on a square.

2. Problem setup

Let a dynamical system with P parameters $\mu = (\mu_1, \ldots, \mu_P) \in \Pi \subset \mathbb{R}^P$ be of the form

$$\begin{aligned} C(\mu)\,\dot{x}(t,\mu) &= G(\mu)\,x(t,\mu) + B(\mu)\,u(t) \\ y(t,\mu) &= L(\mu)\,x(t,\mu), \end{aligned} \tag{1}$$

which is linear in state. The state-vector is denoted by $x(t,\mu) \in \mathbb{R}^N$, whereas $u(t) \in \mathbb{R}^M$ and $y(t,\mu) \in \mathbb{R}^K$ represent the M inputs and the K outputs (quantities of interest), respectively, in the underlying model. We call the size N of the state-vector the dimension of the parametric model (1). The system matrices $C(\mu), G(\mu) \in \mathbb{R}^{N\times N}$, $B(\mu) \in \mathbb{R}^{N\times M}$ and $L(\mu) \in \mathbb{R}^{K\times N}$ are parameter-dependent. We assume that the matrices $C(\mu)$ are regular and that the system (1) is asymptotically stable for each $\mu \in \Pi$, meaning that all eigenvalues of $C(\mu)^{-1}G(\mu)$ have negative real parts (see [13,3]). The parameter domain Π is usually bounded, for example, a compact cuboid. The dimension of the dynamical system is commonly very large, and consequently simulations for many different parameter values or varying input functions may become too expensive.

The aim of parametric model reduction is to find a dynamical system

$$\begin{aligned} C_{\text{red}}(\mu)\,\dot{z}(t,\mu) &= G_{\text{red}}(\mu)\,z(t,\mu) + B_{\text{red}}(\mu)\,u(t) \\ y_{\text{red}}(t,\mu) &= L_{\text{red}}(\mu)\,z(t,\mu) \end{aligned} \tag{2}$$

of dimension $n \ll N$ with $y_{\text{red}}(t,\mu) \approx y(t,\mu)$ for all $t \geq 0$, all $p \in \Pi$ and a broad class of inputs u. This means finding a dynamical system of much lower dimension which has a similar input–output-behavior as the original system and preserves the parameter dependency.

To study the input–output-behavior of a linear dynamical system we consider the *transfer function* of (1) in the frequency domain, cf. [13] for the non-parametric case,

$$H(s,\mu) := L(\mu)\;(s\,C(\mu) - G(\mu))^{-1}\;B(\mu) \in \mathbb{C}^{K\times M},$$

where $s \in S(\mu) \subseteq \mathbb{C}$ with $S(\mu) := \{s \in \mathbb{C} \;:\; \det(s\,C(\mu) - G(\mu)) \neq 0\}$.

The aim of our parametric model reduction is to find a system of the form (2) with

$$H_{\text{red}}(s,\mu) \approx H(s,\mu)$$

for all parameter values $\mu \in \Pi$ and a broad range of frequency points $s \in \mathbb{C}$, where H_{red} denotes the transfer function of the reduced system (2). We may achieve this approximation by using two (global) projection matrices $V, W \in \mathbb{C}^{N\times n}$ and the projected matrices

$$\begin{aligned} C_{\text{red}}(\mu) &= W^{\text{H}}C(\mu)V, \qquad & G_{\text{red}}(\mu) &= W^{\text{H}}G(\mu)V, \\ B_{\text{red}}(\mu) &= W^{\text{H}}B(\mu), \qquad & L_{\text{red}}(\mu) &= L(\mu)V. \end{aligned} \tag{3}$$

Unfortunately, appropriate matrices V and W cannot be computed directly for all parameter values at once. To obtain matrices V and W that apply well for a range of parameters methods for reduced order models of non-parametric systems are needed.

3. Methods for model order reduction

In this section, we briefly review approaches to compute reduced order models of non-parametric systems. These techniques will be used within a pMOR to build the global projection matrices V and W for the projections (3).

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