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New third- and fourth-order singly diagonally implicit two-step peer triples with local and global error controls for solving stiff ordinary differential equations

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Abstract

In this paper we present new singly diagonally implicit two-step peer triples equipped with local and global error controls for providing preassigned accuracies of numerical integration of stiff ordinary differential equations (ODEs) in automatic mode. Recently, Kulikov and Weiner [A singly diagonally implicit two-step peer triple with global error control for stiff ordinary differential equations, *SIAM J. Sci. Comput.*, **2015**, 37(3), A1593–A1613] reported an efficient numerical integration tool of order 2, which solves accurately many difficult stiff ODEs, including large-scale systems obtained from semidiscretization of partial differential equations (PDEs), corresponding to user-supplied requests. Moreover, the cited method is not only accurate, but it is also more efficient than the well-known Matlab code `ODE23s` with local error control. Here, we further extend the published technique and construct variable-stepsize singly diagonally implicit two-step peer triples of the higher orders 3 and 4. Our numerical experiments suggest that these triples are suitable for treating stiff problems with prescribed accuracy conditions. In addition, performance of the presented methods can be comparable to the built-in stiff Matlab code `ODE15s` with local error control, which is considered to be a benchmark means for solving stiff ODEs by many practitioners, for some test problems.

Keywords: Ordinary differential equation, stiff problem, singly diagonally implicit two-step peer method, absolute and scaled local and global error estimations, automatic local and global error controls
2000 MSC: 65L05, 65L06, 65L20, 65L50, 65L70.

1. Introduction

Numerical integration tools for solving ordinary differential equations (ODEs) of the form

$$x'(t) = g\big(t, x(t)\big), \quad t \in [t_0, t_{end}], \quad x(t_0) = x^0, \tag{1}$$

where $x(t) \in \mathbb{R}^m$ and $g : D \subset \mathbb{R}^{m+1} \to \mathbb{R}^m$ is a sufficiently smooth function, constitute an important topic in computational mathematics [1–9]. This is because problem (1) arises broadly in

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