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Note on multi-degree reduction of Bézier curves via modified Jacobi–Bernstein basis transformation☆

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Abstract

Recently, Bhrawy et al. [1] proposed a novel method for $C^{b-1,a-1}$-constrained degree reduction of Bézier curves by using the basis transformations between the modified Jacobi polynomials (MJPs) and Bernstein polynomials. In this note, we provide corrections to the formulas wrongly presented in their article. Then we propose $G^{b-1,a-1}$-constrained degree reduction of Bézier curves using MJPs, which can always produce more satisfactory results.

Keywords: Bézier curve; Degree reduction; Modified Jacobi polynomials; Geometric continuity

1. Introduction

Recently, Bhrawy et al. [1] proposed a novel method, called *MJP method*, for $C^{b-1,a-1}$-constrained degree reduction of Bézier curves by using the basis transformations between the modified Jacobi polynomials (MJPs) and Bernstein polynomials. Compared to the previous methods [2, 3], the MJP method is more efficient since it requires fewer computation steps.

For a degree-n Bézier curve $P_n(x) = \sum_{i=0}^{n} p_i B_i^n(x)$, $x \in [0,1]$, defined by the control points $p_i \in \mathbb{R}^d$ and the Bernstein polynomials $B_i^n(x) = \binom{n}{i}(1-x)^{n-i}x^i$, the degree reduction problem aims to find a lower degree Bézier curve $Q_m(x) = \sum_{i=0}^{m} q_i B_i^m(x)$, $x \in [0,1]$, such that the weighted L_2 error ε is minimized,

$$\varepsilon = \langle P_n, Q_m \rangle_\omega := \int_0^1 (1-x)^\alpha x^\beta \left\| P_n(x) - Q_m(x) \right\|^2 dx \tag{1}$$

where $\omega^{\alpha,\beta}(x) = (1-x)^\alpha x^\beta$, $\alpha, \beta > -1$, is the weight function. With specified $a, b \in \mathbb{N}_0$, the MJP method preserves C^{b-1} and C^{a-1} continuity at the two endpoints [1].

The purpose of this note is twofold:

- We provide corrections to the formulas wrongly presented in [1]. In addition, we point out that a unreasonable conclusion was made in [1] when comparing the MJP method with previous methods [4, 5].
- We extend the MJP method to the case of geometric continuity and propose $G^{b-1,a-1}$-constrained degree reduction of Bézier curves using MJPs. As shown in Section 3, more satisfactory results can always be obtained by the new method.

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