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PROJECTED NONSYMMETRIC ALGEBRAIC RICCATI EQUATIONS AND REFINING ESTIMATES OF INVARIANT AND DEFLATING SUBSPACES

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Abstract. We consider the numerical solution of the projected nonsymmetric algebraic Riccati equations or their associated Sylvester equations via Newton's method, arising in the refinement of estimates of invariant (or deflating subspaces) for a large and sparse real matrix A (or pencil $A - \lambda B$). The engine of the method is the inversion of the matrix $P_2P_2^\top A - \gamma I_n$ or $P_{l2}P_{l2}^\top(A - \gamma B)$, for some orthonormal P_2 or P_{l2} from $\mathbb{R}^{n\times(n-m)}$, making use of the structures in A or $A - \lambda B$ and the Sherman-Morrison-Woodbury formula. Our algorithms are efficient, under appropriate assumptions, as shown in our error analysis and illustrated by numerical examples.

Key words. Deflating subspace, invariant subspace, large-scale problem, Newton's method, nonsymmetric algebraic Riccati equation, sparse matrix, Sherman-Morrison-Woodbury formula, Sylvester equation

AMS subject classifications. 15A18, 15A22, 15A24, 65F15, 65F50

1. Introduction. We consider the numerical solution of the projected nonsymmetric algebraic Riccati equation (pNARE) with constraint [5, 19, 21]:

$$\mathcal{N}(\widetilde{R}) \equiv P_2P_2^\top A\widetilde{R} - \widetilde{R}A_{11} - \widetilde{R}P_1^\top A\widetilde{R} + P_2P_2^\top AP_1 = 0, \quad P_1^\top \widetilde{R} = 0, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$, $\widetilde{R} \in \mathbb{R}^{n\times m}$, $P \equiv [P_1,\, P_2] \in \mathbb{R}^{n\times n}$ is orthogonal and $A_{11} \equiv P_1^\top AP_1 \in \mathbb{R}^{m\times m}$. We are particularly interested in the case when A is large and structured, as in the refinement of invariant subspaces in Section 1.1. Notice the constraint $P_1^\top \widetilde{R} = 0$ in (1.1).

We also consider the numerical solution of the projected coupled nonsymmetric algebraic Riccati equation (pCNARE) with constraint:

$$\widetilde{\mathcal{N}}(\widetilde{L}, \widetilde{R}) \equiv \begin{bmatrix} P_{l2}P_{l2}^\top A\widetilde{R} - \widetilde{L}A_{11} - \widetilde{L}P_{l1}^\top A\widetilde{R} + P_{l2}P_{l2}^\top AP_{r1} \\ P_{l2}P_{l2}^\top B\widetilde{R} - \widetilde{L}B_{11} - \widetilde{L}P_{l1}^\top B\widetilde{R} + P_{l2}P_{l2}^\top BP_{r1} \end{bmatrix} = 0, \quad \begin{bmatrix} P_{l1}^\top \widetilde{L} \\ P_{r1}^\top \widetilde{R} \end{bmatrix} = 0, \tag{1.2}$$

where $A, B \in \mathbb{R}^{n\times n}$, $\widetilde{L}, \widetilde{R} \in \mathbb{R}^{n\times m}$, $P_l \equiv [P_{l1},\, P_{l2}], P_r \equiv [P_{r1},\, P_{r2}] \in \mathbb{R}^{n\times n}$ are orthogonal and $A_{11} \equiv P_{l1}^\top AP_{r1}, B_{11} \equiv P_{l1}^\top BP_{r1} \in \mathbb{R}^{m\times m}$. We are particularly interested in the case when A and B are large and structured, as in the refinement of deflating subspaces in Section 3.

The pNARE (1.1) arises from the standard eigenvalue problem (SEP) $Ax = \lambda x$ $(x \neq 0)$ and the pCNARE (1.2) from the generalized eigenvalue problem (GEP) $Ax = \lambda Bx$ $(x \neq 0)$ (see Section 3 for details). For moderate values of n, the well-known QR or QZ algorithm [13] solves the SEP or GEP satisfactorily. For large values of n, especially when A, B possess appropriate structures and sparsity, the subspace or Arnoldi type iterative methods may be applied. For the SEP, we obtain the estimates of some invariant subspace $\mathcal{X}$, that is $A\mathcal{X} \subseteq \mathcal{X}$. Columns of the orthonormal $P_1 \in \mathbb{R}^{n\times m}$ span the estimate of $\mathcal{X}$, with $A_{11} \equiv P_1^\top AP_1$ containing the estimates of the corresponding eigenvalues. For the GEP, the corresponding methods produce the estimates of some deflating subspaces $(\mathcal{X}, \mathcal{Y})$, that is $A\mathcal{X} \subseteq \mathcal{Y}, B\mathcal{X} \subseteq \mathcal{Y}$. Let the columns in the $n \times m$ real orthonormal P_{r1} and P_{l1}, with $P_{r1}^\top P_{r1} = I_m = P_{l1}^\top P_{l1}$, span the estimates of $\mathcal{X}$ and $\mathcal{Y}$ respectively. The estimates of the corresponding eigenvalues are then the eigenvalues of the matrix pencil $A_{11} - \lambda B_{11} \equiv P_{l1}^\top(A - \lambda B)P_{r1}$.

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