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The WR-HSS iteration method for a system of linear differential equations and its applications to the unsteady discrete elliptic problem *

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Abstract

We consider the numerical method for non-self-adjoint positive definite linear differential equations, and its application to the unsteady discrete elliptic problem, which is derived from spatial discretization of the unsteady elliptic problem with Dirichlet boundary condition. Based on the idea of the alternating direction implicit (ADI) iteration technique and the Hermitian/skew-Hermitian splitting (HSS), we establish a waveform relaxation (WR) iteration method for solving the non-self-adjoint positive definite linear differential equations, called the WR-HSS method. We analyze the convergence property of the WR-HSS method, and prove that the WR-HSS method is unconditionally convergent to the solution of the system of linear differential equations. In addition, we derive the upper bound of the contraction factor of the WR-HSS method in each iteration which is only dependent on the Hermitian part of the corresponding non-self-adjoint positive definite linear differential operator. Finally, the applications of the WR-HSS method to the unsteady discrete elliptic problem demonstrate its effectiveness and the correctness of the theoretical results.

Keywords: GMRES; HS splitting; SOR; system of linear equations; unsteady discrete elliptic problem; waveform relaxation.

1 Introduction

We consider the numerical solution of the following unsteady elliptic problem (second-order parabolic equation),

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} - \nabla \cdot [a(x,t)\nabla u(x,t)] + \sum_{j=1}^{d} \frac{\partial}{\partial x_j}(p(x,t)u(x,t)) = q(x,t),\ x \in \Omega,\ t \in [0,\mathrm{T}], \\ u(x,0) = u_0(x),\ x \in \Omega, \\ u(x,t) = v(x,t),\ x \in \partial\Omega,\ t \in [0,\mathrm{T}] \end{cases} \tag{1.1}$$

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