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Estimation of the inverse Weibull parameters under adaptive type-II progressive hybrid censoring scheme

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ABSTRACT

This paper describes the frequentist and Bayesian estimation for the scale parameter λ and shape parameter β of the inverse Weibull (IW) distribution based on adaptive type-II progressive hybrid censoring scheme (AT-II PHCS). We discuss the maximum likelihood estimators (MLEs) and the approximate MLEs, where the MLEs cannot be obtained in closed forms. The Bayes estimates for the IW parameters are obtained based on squared error (SE) loss function by using the approximation form of Lindley (1980). The optimal censoring scheme has been suggested using two different optimality criteria. A real life data set is used for illustration purpose. Finally, the different proposed estimators have been compared through an extensive simulation studies.

Contents

1. Introduction

If the random variable Y has a Weibull distribution, then the random variable $X = Y^{-1}$ has an IW distribution with probability density function (pdf), cumulative distribution function (cdf) and hazard rate function respectively given by

$$f(x) = \lambda\beta x^{-(\beta+1)} e^{-\lambda x^{-\beta}}, \quad x > 0,\ \lambda,\ \beta > 0, \tag{1}$$

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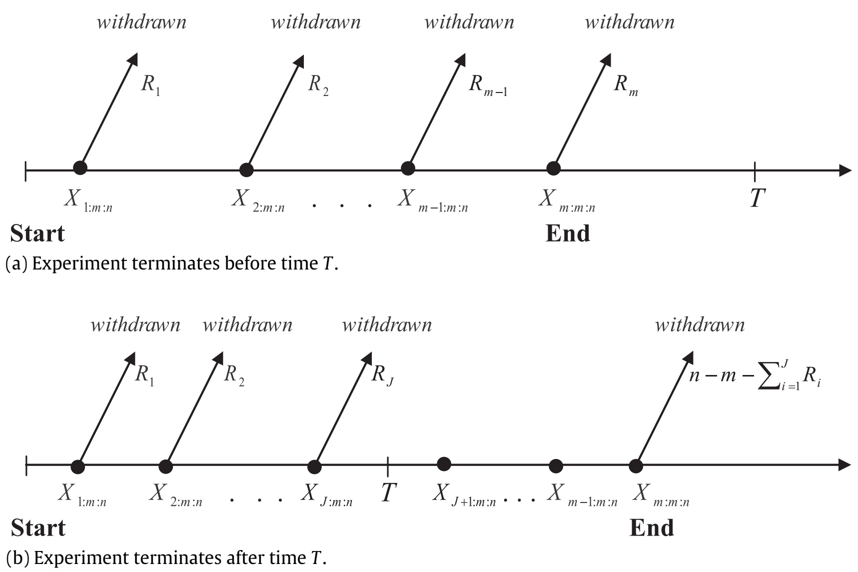

Fig. 1. Schematic representation of AT-II PHCS.

and

$$F(x) = e^{-\lambda x^{-\beta}}, \tag{2}$$

and

$$h(x) = \lambda\beta x^{-(\beta+1)} \left(e^{\lambda x^{-\beta}} - 1\right)^{-1}.$$

The IW distribution is more appropriate model than the Weibull distribution because the Weibull distribution does not provide a satisfactory parametric fit if the data indicate a non-monotone and unimodal hazard rate functions. The hazard rate function of IW distribution can be decreasing or increasing depending on the value of the shape parameter. The IW distribution is useful to model several data such as the time to breakdown of an insulating fluid subjected to the action of a constant tension and degradation of mechanical components such as pistons and crankshafts of diesel engines. Extensive work has been done on the IW distribution, see for example, [1–4] and for more details about the generalizations of IW distribution see [5]. In addition, many articles have considered IW distribution under different censoring schemes. Among others, Kundu and Howlader [6], Musleh and Helu [7], Sultan et al. [8] and Xiuyun and Zaizai [9].

Progressive hybrid censoring scheme in the context of life testing experiments was introduced by Kundu and Joarder [10]. They considered a type-I progressive hybrid censoring scheme, in which n identical units are placed on test with predetermined progressive censoring scheme $R_1, R_2, \ldots, R_m$. At the time of the first failure $x_{1:m:n}$, R_1 units are randomly removed from the remaining $n-1$ surviving units. Similarly, at the time of the second failure $x_{2:m:n}$, R_2 units of the remaining $n-2-R_1$ units are randomly removed and so on. The experiment is terminated at random time $T^* = \min\{x_{m:m:n}, T\}$ where $x_{m:m:n}$ is the mth failure and $T \in (0, \infty)$ is a predetermined time. For more details see also [11]. The drawback of the type-I progressive hybrid censoring scheme is that the effective sample size is random and it can turn out to be a very small number, therefore, the statistical inference procedure may not be applicable or will have low efficiency. Ng et al. [12] introduced an AT-II PHCS to increase the efficiency of statistical analysis and save the total test time and analyzed the data under the assumption of exponential distribution. In AT-II PHCS the effective number of failures m is fixed in advance and the experimental time is allowed to run over time T which is an ideal total test. In this case, the progressive censoring scheme $R_1, R_2, \ldots, R_m$ is provided, but the values of some of the R_i may change accordingly during the experiment. If the mth progressively censored observed failures occurs before time T (i.e. $X_{m:m:n} < T$), the experiment stops at this time $X_{m:m:n}$, and we will have a usual type-II progressive censoring scheme with the pre-fixed progressive censoring scheme $R_1, R_2, \ldots, R_m$ (Fig. 1(a)). Otherwise, if $X_{J:m:n} < T < X_{J+1:m:n}$, where $J+1 < m$ and $X_{J:m:n}$ is the Jth failure time occur before time T, then we will not withdraw any items from the experiment by setting $R_{J+1}, R_{J+2}, \ldots, R_{m-1} = 0$ and at the time of the mth failure all remaining surviving items are removed, i.e., $R_m = n - m - \sum_{i=1}^{J} R_i$ (Fig. 1(b)). The main advantage of this scheme is to speed up the experiment when the experiment duration exceed the predetermined time T and assures us to get the effective number of failures m.

Many authors have considered AT-II PHCS. Lin et al. [13], discussed the maximum likelihood and approximate maximum likelihood estimators for the Weibull distribution. Hemmati and Khorram [14], studied the maximum likelihood and approximate maximum likelihood estimators for the log-normal distribution. Mahmoud et al. [15], investigated the maximum likelihood and Bayes estimates of the unknown parameters of Pareto distribution. Ashour and Nassar [16] showed