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On the discretization and application of two space–time boundary integral equations for 3D wave propagation problems in unbounded domains ☆

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A B S T R A C T

In this paper, we consider 3D wave propagation problems in unbounded domains, such as those of acoustic waves in non viscous fluids, or of seismic waves in (infinite) homogeneous isotropic materials, where the propagation velocity c is much higher than 1. For example, in the case of air and water $c \approx 343\,\mathrm{m/s}$ and $c \approx 1500\,\mathrm{m/s}$ respectively, while for seismic P-waves in linear solids we may have $c \approx 6000\,\mathrm{m/s}$ or higher. These waves can be generated by sources, possible away from the obstacles. We further assume that the dimensions of the obstacles are much smaller than that of the wave velocity, and that the problem transients are not excessively short.

For their solution we consider two different approaches. The first directly uses a known space–time boundary integral equation to determine the problem solution. In the second one, after having defined an artificial boundary delimiting the region of computational interest, the above mentioned integral equation is interpreted as a non reflecting boundary condition to be coupled with a classical finite element method.

For such problems, we show that in some cases the computational cost and storage, required by the above numerical approaches, can be significantly reduced by taking into account a property that till now has not been considered. To show the effectiveness of this reduction, the proposed approach is applied to several problems, including multiple scattering.

1. Introduction

In the last decades, space–time Boundary Integral Equations (BIE) have been successfully applied to wave propagation problems defined in the exterior of a bounded domain (see, for example, [9], [24], [3], [19], [26], [15], [10], [2], [27], [16], [5], [4], [25], [18], [17], [22], [12]).

Most of them, however, are for homogeneous problems with trivial initial values. Furthermore, they are generally used to determine the problem solution at chosen points. Only in the last few years (see [13], [2], [14], [6]), a BIE for the classical wave equation has been used to define a Non Reflecting Boundary Condition (NRBC) on a chosen artificial boundary,

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surrounding the computational domain. Its discretization is then coupled with that of the domain of interest by means of finite elements or finite differences.

In the case of the classical 3D non homogeneous wave equation, the problem we consider in this paper is the following:

$$\begin{cases} \frac{1}{c^2}u^e_{tt}(\mathbf{x},t) - \Delta u^e(\mathbf{x},t) & = f(\mathbf{x},t) \quad \text{in } \Omega^e \times (0,T) \\ u^e(\mathbf{x},t) & = g(\mathbf{x},t) \quad \text{in } \Gamma \times (0,T) \\ u^e(\mathbf{x},0) & = u_0(\mathbf{x}) \quad \text{in } \Omega^e \\ u^e_t(\mathbf{x},0) & = v_0(\mathbf{x}) \quad \text{in } \Omega^e \end{cases} \tag{1}$$

where $\Omega^e = \mathbb{R}^3 \setminus \overline{\Omega^i}$, Ω^i being a bounded open domain, having a smooth boundary Γ, or the union $\cup_{k=1}^{\mathcal{K}} \Omega^i_k$ of well separated domains of this type. As often occurs in practical situations, we assume that the initial values u_0, v_0 and the source term f have local supports. Furthermore, we also assume that these data satisfy the smoothness and compatibility conditions which guarantee a sufficiently smooth solution u^e, as required (see [15], [24]) by the numerical approach we will describe in the next section.

To solve problem (1) we will consider the following single–double layer BIE (see [14]):

$$\frac{1}{2}u^e(\mathbf{x},t) - (\mathcal{V}\partial_{\mathbf{n}}u^e)(\mathbf{x},t) + (\mathcal{K}u^e)(\mathbf{x},t) = I_{u_0}(\mathbf{x},t) + I_{v_0}(\mathbf{x},t) + I_f(\mathbf{x},t) \quad \mathbf{x} \in \Gamma, \tag{2}$$

with

$$\mathcal{V}\psi(\mathbf{x},t) := \int_0^t \int_\Gamma G(\mathbf{x}-\mathbf{y},t-\tau)\,\psi(\mathbf{y},\tau)\,d\Gamma_{\mathbf{y}}\,d\tau = \int_\Gamma \frac{\psi(\mathbf{y},t-\|\mathbf{x}-\mathbf{y}\|/c)}{4\pi\,\|\mathbf{x}-\mathbf{y}\|}\,d\Gamma_{\mathbf{y}} \tag{3}$$

and

$$\mathcal{K}\varphi(\mathbf{x},t) := \int_0^t \int_\Gamma \partial_{\mathbf{n}}G(\mathbf{x}-\mathbf{y},t-\tau)\,\varphi(\mathbf{y},\tau)\,d\Gamma_{\mathbf{y}}\,d\tau = \int_\Gamma \partial_{\mathbf{n}}\frac{\varphi(\mathbf{y},t-\|\mathbf{x}-\mathbf{y}\|/c)}{4\pi\,\|\mathbf{x}-\mathbf{y}\|}\,d\Gamma_{\mathbf{y}}. \tag{4}$$

The last expressions in (3) and (4) have been obtained by interchanging (see [15]) the time and space integrals in the corresponding representations, and using the wave equation fundamental solution expression

$$G(\mathbf{x},t) = \frac{\delta(t-\|\mathbf{x}\|/c)}{4\pi\,\|\mathbf{x}\|}, \tag{5}$$

where $\delta(\cdot)$ is the well known Dirac delta function. The symbol $\partial_{\mathbf{n}} = \partial_{\mathbf{n}_{\mathbf{y}}}$ denotes the outward unit normal (distributional) derivative, with respect to the $\mathbf{y}$-variable, on the boundary Γ.

The "volume" terms I_{u_0}, I_{v_0} and I_f are generated by the non homogeneous initial conditions and the non trivial source, respectively. These volume terms have the following integral representations (see [15]):

$$I_{u_0}(\mathbf{x},t) = \frac{\partial}{\partial t}\int_{\operatorname{supp}(u_0)} G(\mathbf{x}-\mathbf{y},t)\,u_0(\mathbf{y})\,d\mathbf{y}, \tag{6}$$

$$I_{v_0}(\mathbf{x},t) = \int_{\operatorname{supp}(v_0)} G(\mathbf{x}-\mathbf{y},t)\,v_0(\mathbf{y})\,d\mathbf{y}, \tag{7}$$

$$I_f(\mathbf{x},t) = \int_0^t \int_{\operatorname{supp}(f)} G(\mathbf{x}-\mathbf{y},t-\tau)\,f(\mathbf{y},\tau)\,d\mathbf{y}\,d\tau. \tag{8}$$

The mapping properties of the above operators $\mathcal{V}$, $\mathcal{K}$, when these are acting in proper functional spaces, are well-known; see, for example, [24], [21], [28]. In particular, for any real $r \geq 0$,

$$\mathcal{V} : H_0^{r+1}(0,T;H^{-1/2}(\Gamma)) \to H_0^r(0,T;H^{1/2}(\Gamma)) \tag{9}$$

and

$$\mathcal{K} : H_0^{r+3/2}(0,T;H^{1/2}(\Gamma)) \to H_0^r(0,T;H^{1/2}(\Gamma)) \tag{10}$$

are bounded.

The above spaces are defined as follows. Set first $H_0^r(0,T) = \{g_{|(0,T)} : g \in H^r(\mathbb{R}) \text{ with } g \equiv 0 \text{ on } (-\infty,0)\}$, where H^r denotes the classical Sobolev space of order r. When r is an integer, this space consists of those functions g whose r-th distributional derivative is in $L^2(0,T)$ and which have $g(0) = \ldots g^{(r-1)}(0) = 0$. Then: