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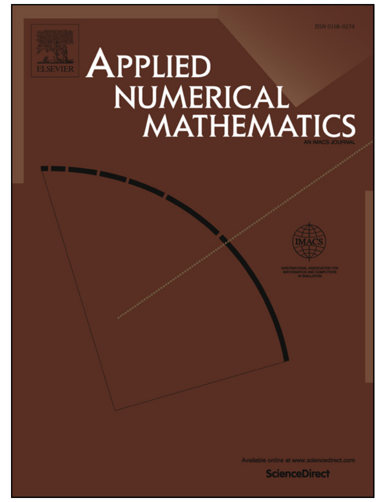

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Algorithms for the implementation of adaptive isogeometric methods using hierarchical B-splines

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Abstract

In this article we introduce all the ingredients to develop adaptive isogeometric methods based on hierarchical B-splines. In particular, we give precise definitions of local refinement and coarsening that, unlike previously existing methods, can be understood as the inverse of each other. We also define simple and intuitive data structures for the implementation of hierarchical B-splines, and algorithms for refinement and coarsening that take advantage of local information. We complete the paper with some simple numerical tests to show the performance of the data structures and algorithms, that have been implemented in the open-source Octave/Matlab code GeoPDEs.

Keywords: Isogeometric analysis, adaptive methods, hierarchical splines, local refinement, coarsening

1. Introduction

The use of high order spline spaces for the numerical discretization of partial differential equations has been increased and spread due to the emergence of the isogeometric analysis (IGA) techniques [1, 2]. Introduced originally to enhance the interoperability between computer aided design (CAD) and finite element softwares, the main idea behind IGA is to use B-splines or rational B-splines (NURBS) functions, which are the standard in CAD, both for geometry representation and for the computation of the discrete solution to the equation. A state-of-the-art review on the existing mathematical results can be found in [3].

One of the most active research topics on IGA is the development of adaptive methods for local refinement and coarsening. These methods require the use of spline spaces that go beyond the tensor product structure of B-splines [4, 5], and several alternatives have been already proposed and tested in the IGA community, such as hierarchical B-splines, T-splines, LR-splines or PHT-splines. Among them, hierarchical B-splines are probably the easiest to define and to implement for their use in IGA [6], thanks to their multilevel structure: first one introduces a sequence of B-spline spaces of different levels and a sequence of subdomains, which determine the hierarchical mesh, then the set of active functions in each level is decided by a simple check on their support compared to the subdomains. This simplicity, together with the flexibility in the choice of the refined subdomains, has favored their application by many different research groups, see for example [7, 8, 9, 10].

However, when facing the implementation of adaptive IGA methods with hierarchical B-splines, most of the works we are aware of [11, 12, 13, 14] fail to reflect this simplicity and/or flexibility in their algortihms and data structures. Entering into the details, the data structures in [11] constrain to refine on disjoint (closed) hyperrectangular regions, which is not flexible enough for adaptive refinement. The data structures in [12] are probably the clearest ones, but their algorithms neglect local information for the update of active functions and elements, in practice requiring a global recomputation of the data structures even when local

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