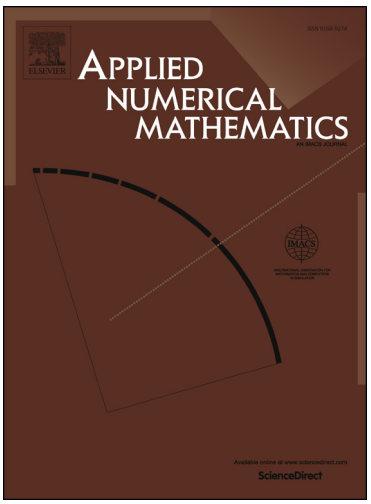
Accepted Manuscript



Supercloseness of the continuous interior penalty method for singularly perturbed problems in 1D: vertex-cell interpolation

Jin Zhang, Xiaowei Liu

PII: S0168-9274(17)30184-8
DOI: http://dx.doi.org/10.1016/j.apnum.2017.09.003
Reference: APNUM 3252

To appear in: *Applied Numerical Mathematics*

Received date: 9 June 2017
Revised date: 11 September 2017
Accepted date: 11 September 2017

Please cite this article in press as: J. Zhang, X. Liu, Supercloseness of the continuous interior penalty method for singularly perturbed problems in 1D: vertex-cell interpolation, *Appl. Numer. Math.* (2017), http://dx.doi.org/10.1016/j.apnum.2017.09.003

Supercloseness of the continuous interior penalty method for singularly perturbed problems in 1D: vertex-cell interpolation ☆

Jin Zhang[a,*], Xiaowei Liu[b,1]

[a] School of Mathematics and Statistics, Shandong Normal University, Jinan 250014, China
[b] College of Science, Qilu University of Technology, Jinan 250353, China

Abstract

A continuous interior penalty method with piecewise polynomials of degree $p \geq 2$ is applied on a Shishkin mesh to solve a singularly perturbed convection–diffusion problem, whose solution has a single boundary layer. This method is analyzed by means of a series of integral identities developed for the convection terms. Then we prove a supercloseness bound of order 5/2 for a vertex-cell interpolation when $p = 2$. The sharpness of our analysis is supported by some numerical experiments. Moreover, numerical tests show supercloseness clearly for $p \geq 3$.

Keywords: Singular perturbation, Convection–diffusion equation, Continuous interior penalty method, Shishkin mesh, Supercloseness

1. Introduction

We consider the following two-point boundary value problem

$$\begin{aligned} -\varepsilon u'' + b(x)u' + c(x)u &= f(x) \quad \text{in } \Omega := (0,1), \\ u(0) = u(1) &= 0, \end{aligned} \tag{1}$$

with a positive parameter ε and sufficiently smooth functions b, c and f. There exist constants β and μ such that

$$b(x) > \beta > 0, \quad c(x) - \frac{1}{2}b'(x) \geq \mu > 0 \quad \text{for } x \in [0,1], \tag{2}$$

☆ Jin Zhang is supported by National Science Foundation of China (11771257), Shandong Provincial Natural Science Foundation, China (ZR2017MA003) and A Project of Shandong Province Higher Educational Science and Technology Program (J17KA169); Xiaowei Liu is supported by National Science Foundation of China (11601251), Shandong Provincial Natural Science Foundation, China (ZR2016AM13) and A Project of Shandong Province Higher Educational Science and Technology Program (J16LI10).

* Corresponding author: jinzhangalex@hotmail.com
[1] Email: xwliuvivi@hotmail.com