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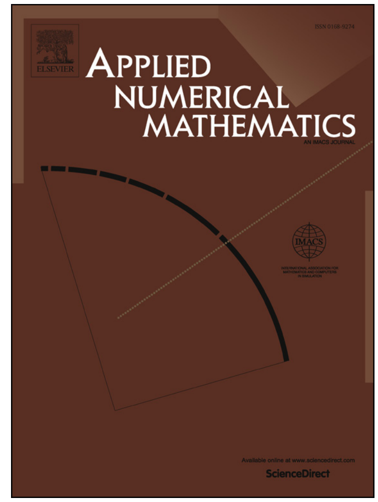

Solving a class of nonlinear boundary integral equations based on the meshless local discrete Galerkin (MLDG) method

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Solving a class of nonlinear boundary integral equations based on the meshless local discrete Galerkin (MLDG) method

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Abstract. The main purpose of this article is to investigate a computational scheme for solving a class of nonlinear boundary integral equations which occurs as a reformulation of boundary value problems of Laplace's equations with nonlinear Robin boundary conditions. The method approximates the solution by the Galerkin method based on the use of moving least squares (MLS) approach as a locally weighted least square polynomial fitting. The discrete Galerkin method for solving boundary integral equations results from the numerical integration of all integrals appeared in the method. The numerical scheme developed in the current paper utilizes the non-uniform Gauss-Legendre quadrature rule to estimate logarithm-like singular integrals. Since the proposed method is constructed on a set of scattered points, it does not require any background mesh and so we can call it as the meshless local discrete Galerkin (MLDG) method. The scheme is simple and effective to solve boundary integral equations and its algorithm can be easily implemented. We also obtain the error bound and the convergence rate of the presented method. Finally, numerical examples are included to show the validity and efficiency of the new technique and confirm the theoretical error estimates.

MSC (2010): 65D10; 45G05; 74S25; 65G99

Keywords: Nonlinear boundary integral equation, Laplace's equation, Discrete Galerkin method, Moving least squares (MLS) method, Meshless Method, Error analysis

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1 Introduction

Consider the following nonlinear boundary integral equation:

$$-\pi u(\mathbf{x})+\int_{\partial D} u(\mathbf{y})\frac{\partial \ln\|\mathbf{x}-\mathbf{y}\|}{\partial n_{\mathbf{y}}}\mathbf{d}s_{\mathbf{y}}+\int_{\partial D} g(\mathbf{y},u(\mathbf{y}))\ln\|\mathbf{x}-\mathbf{y}\|\mathbf{d}s_{\mathbf{y}}=\int_{\partial D} f(\mathbf{y})\ln\|\mathbf{x}-\mathbf{y}\|\mathbf{d}s_{\mathbf{y}},\quad \mathbf{x}\in\partial D. \quad (1)$$

where D is a bounded, open, simply connected region in the plane and its boundary is denoted by ∂D, $n_{\mathbf{x}}$ is the outward unit normal on the boundary ∂D, $f(\mathbf{x})$ is a given function on ∂D, the known function $g(\mathbf{x},u)$ is assumed to be continuous on $\partial D\times\mathbb{R}$ and the unknown function $u(\mathbf{x})\in C^1(\bar{D})\cap C^2(D)$ must be determined. These types of integral equations deduce from reformulations of the boundary value problem for two-dimensional Laplace's equation with nonlinear Robin boundary conditions [11, 29] which arises in various branches of applied science such as solid and fluid mechanics, electrostatics, heat transfer, diffraction and scattering of waves, etc [16, 18, 21, 49].

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