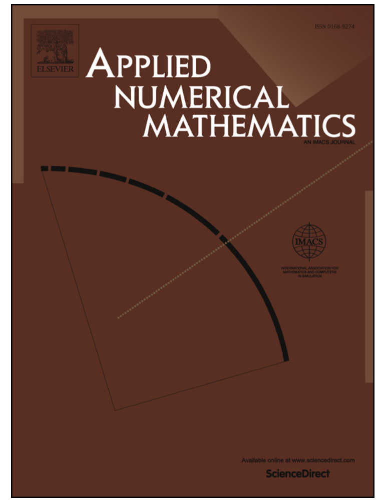


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# Recognition of a time-dependent source in a time-fractional wave equation

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**Abstract**

In the present paper, we deal with an inverse source problem for a time-fractional wave equation in a bounded domain in $\mathbb{R}^d$. The time-dependent source is determined from an additional measurement in the form of integral over the space subdomain. The existence, uniqueness and regularity of a weak solution are obtained. A numerical algorithm based on Rothe's method is proposed, a priori estimates are proved and convergence of iterates towards the solution is established.



*Keywords:* time-fractional wave equation, inverse source problem, reconstruction, convergence, time discretization

## 1. Introduction

Consider a partial differential equation (PDE) with a fractional derivative in time $t$

$$(g_{2-\beta} * \partial_{tt} u(x))\,(t) - \Delta u(x,t) = h(t) f(x) + F(x,t,u(x,t)), \qquad x \in \Omega,\ t \in (0,T), \tag{1}$$

where $\Omega \subset \mathbb{R}^d$ is a bounded domain with a Lipschitz boundary $\Gamma$ (cf. [15]), $T > 0$, $g_{2-\beta}$ is the Riemann-Liouville kernel given by

$$g_{2-\beta}(t) = \frac{t^{1-\beta}}{\Gamma(2-\beta)}, \qquad t > 0,\ 1 < \beta < 2,$$

and $*$ denotes the convolution on the positive half-line, i.e.

$$(k * v)(t) = \int_0^t k(t-s) v(s)\, \mathrm{d}s.$$

Thus, the Caputo fractional derivative of order $\beta$, cf. e.g. [1, 26], defined by

$$\partial_t^\beta u(x,t) = (g_{2-\beta} * \partial_{tt} u(x))\,(t),$$

appears in the equation (1). Note that the equation (1) is a classical diffusion or wave equation for $\beta = 1$ and $\beta = 2$, respectively. We supplement governing PDE (1) with the following initial and boundary conditions

$$\begin{aligned} u(x,0) &= u_0(x), && x \in \Omega, \\ \partial_t u(x,0) &= v_0(x), && x \in \Omega, \\ -\nabla u(x,t) \cdot \boldsymbol{\nu} &= g(x,t), && (x,t) \in \Gamma \times (0,T), \end{aligned} \tag{2}$$

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