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K. Šišková, M. Slodička

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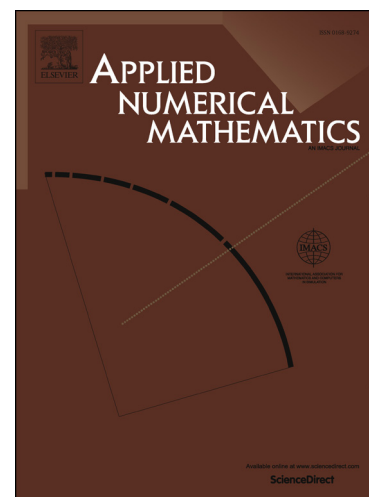
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Recognition of a time-dependent source in a time-fractional wave equation

K. Šišková, M. Slodička

Department of Mathematical Analysis, research group of Numerical Analysis and Mathematical Modeling (NaM²), Ghent University, Belgium

Abstract

In the present paper, we deal with an inverse source problem for a time-fractional wave equation in a bounded domain in \mathbb{R}^d . The time-dependent source is determined from an additional measurement in the form of integral over the space subdomain. The existence, uniqueness and regularity of a weak solution are obtained. A numerical algorithm based on Rothe's method is proposed, a priori estimates are proved and convergence of iterates towards the solution is established.

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Keywords: time-fractional wave equation, inverse source problem, reconstruction, convergence, time discretization

1. Introduction

Consider a partial differential equation (PDE) with a fractional derivative in time t

$$(g_{2-\beta} * \partial_{tt} u(x))(t) - \Delta u(x, t) = h(t)f(x) + F(x, t, u(x, t)), \quad x \in \Omega, \quad t \in (0, T), \quad (1)$$

where $\Omega \subset \mathbb{R}^d$ is a bounded domain with a Lipschitz boundary Γ (cf. [15]), $T > 0$, $g_{2-\beta}$ is the Riemann-Liouville kernel given by

$$g_{2-\beta}(t) = \frac{t^{1-\beta}}{\Gamma(2-\beta)}, \quad t > 0, \quad 1 < \beta < 2,$$

and $*$ denotes the convolution on the positive half-line, i.e.

$$(k * v)(t) = \int_0^t k(t-s)v(s) ds.$$

Thus, the Caputo fractional derivative of order β , cf. e.g. [1, 26], defined by

$$\partial_t^\beta u(x, t) = (g_{2-\beta} * \partial_{tt} u(x))(t),$$

appears in the equation (1). Note that the equation (1) is a classical diffusion or wave equation for $\beta = 1$ and $\beta = 2$, respectively. We supplement governing PDE (1) with the following initial and boundary conditions

$$\begin{aligned} u(x, 0) &= u_0(x), & x \in \Omega, \\ \partial_t u(x, 0) &= v_0(x), & x \in \Omega, \\ -\nabla u(x, t) \cdot \nu &= g(x, t), & (x, t) \in \Gamma \times (0, T), \end{aligned} \quad (2)$$

Email addresses: katarina.siskova@ugent.be (K. Šišková), marian.slodicka@ugent.be (M. Slodička)
URL: <http://cage.ugent.be/~ms> (M. Slodička)

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