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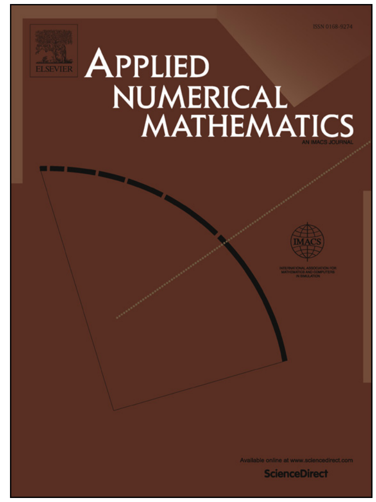

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A posteriori error estimates and adaptivity for the discontinuous Galerkin solutions of nonlinear second-order initial-value problems

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Abstract

In this paper, we propose and analyze an efficient and reliable *a posteriori* error estimator of residual-type for the discontinuous Galerkin (DG) method applied to nonlinear second-order initial-value problems for ordinary differential equations. This estimator is simple, efficient, and asymptotically exact. We use our recent optimal L^2 error estimates and superconvergence results [Baccouch, Appl. Numer. Math 115 (2017) 160–179] to show that the significant parts of the DG discretization errors are proportional to the $(p+1)$-degree right Radau polynomial, when polynomials of total degree not exceeding p are used. These new results allow us to construct a residual-based *a posteriori* error estimator which is obtained by solving a local residual problem with no initial condition on each element. We prove that, for smooth solutions, the proposed *a posteriori* error estimator converges to the actual error in the L^2-norm with order of convergence $p+2$. Computational results indicate that the theoretical order of convergence is sharp. By adding the *a posteriori* error estimate to the DG solution, we obtain a post-processed approximation which superconverges with order $p+2$ in the L^2-norm. Moreover, we demonstrate the effectiveness of the this error estimator. Finally, we present a local adaptive mesh refinement (AMR) procedure that makes use of our local *a posteriori* error estimate. Our proofs are valid for arbitrary regular meshes and for P^p polynomials with $p \geq 1$. Several numerical results are presented to validate the theoretical results.

Keywords: Nonlinear second-order ordinary differential equations; discontinuous Galerkin method; *a posteriori* error estimates; superconvergence; adaptive mesh refinement.

1. Introduction

The main goal of this article is to present and analyze an *a posteriori* error estimator for the discontinuous Galerkin (DG) method applied to the following general nonlinear second-order initial-value problem (IVP)

$$u'' = f(t, u, u'), \quad 0 \leq t \leq T, \quad u(0) = \alpha, \quad u'(0) = \beta, \tag{1.1}$$

where $u : [0,T] \to \mathbb{R}$ and $f : [0,T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a given function. In our analysis, we assume that the function $f(t,u,q)$ satisfies a Lipschitz condition on $D = [0,T] \times \mathbb{R} \times \mathbb{R}$ in