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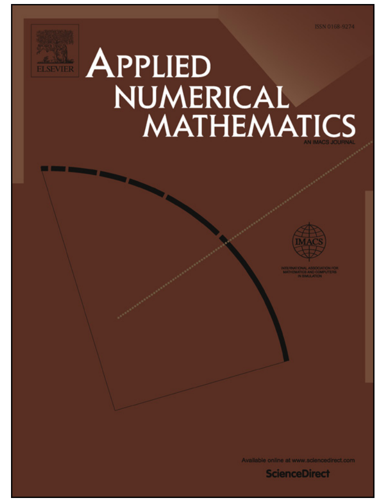

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Curvature-Induced Instability of a Stokes-like Problem with Non-Standard Boundary Conditions

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Abstract

We present an analysis of a set of parametrized boundary conditions for a Stokes-Brinkman model in two space dimensions, discretized by finite elements. We particularly point out an instability which arises when these boundary conditions are posed on a curved line, which then leads to unphysical oscillations. In contrast to a Navier-slip condition, which is prone to Babuška's paradox, this instability can be traced back to the continuous level. We claim that the stability in these cases depend on the amount of curvature at the boundary, which is shown in a reduced setting in cylinder coordinates. The transition to a two dimensional Cartesian case is then based on numerical studies, which further substantiate the claim. Lastly, stabilization techniques are motivated that enhance the continuous FEM setting and are conveniently able to deal with arising oscillations.

Keywords: Stokes equations, boundary conditions, curvature, spurious oscillations, continuous interior penalty, kinetic theory

1. Introduction

This paper discusses an instability that arises in Stokes-like systems with respect to a class of parametrized boundary conditions. Those boundary conditions contain a non-vanishing normal component of the vectorial quantity, also when applied to solid walls. While this setting is unusual in a fluid flow setting, which is the dominant application area of the Stokes equations, it is reasonable in other scenarios. For example, the Stokes problem in our case is a description of heat conduction in a rarefied gas and should be understood as an extension to the Poisson equation for the temperature distribution in the fluid. The equations are obtained as a subsystem of the regularized 13-moment equations (R13), first presented in [1] and extensively discussed in [2]. They are derived from the Boltzmann equation and can be viewed as an extended version of the Navier-Stokes-Fourier (NSF) equations for compressible media with respect to rarefied gas flows. Originated in kinetic theory, these equations are a step towards the idea of deriving continuum models that are valid in regimes that deviate from a so-called local thermodynamic equilibrium. The most striking distinction between NSF and R13 is that R13 comes along with two additional equations for heat flux and stress tensor and a complicated set of boundary conditions, which were first introduced in [3]. The theoretical background of the model lies in the inclusion of microscopic behavior, which starts on

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