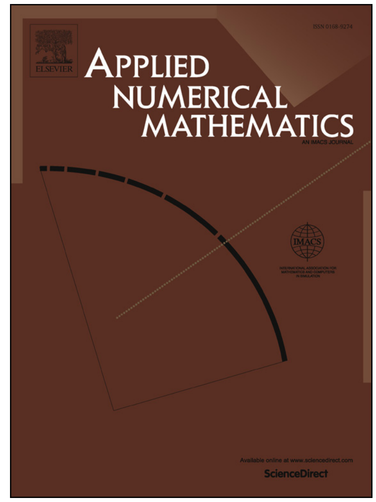
Accepted Manuscript



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PII: S0168-9274(17)30159-9
DOI: http://dx.doi.org/10.1016/j.apnum.2017.07.001
Reference: APNUM 3234

To appear in: *Applied Numerical Mathematics*

Received date: 13 September 2016
Revised date: 23 April 2017
Accepted date: 7 July 2017

Please cite this article in press as: J. Bai et al., Novel alternating update method for low rank approximation of structured matrices, *Appl. Numer. Math.* (2017), http://dx.doi.org/10.1016/j.apnum.2017.07.001

Novel alternating update method for low rank approximation of structured matrices *

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Abstract

This work is devoted to designing a unified alternating update method for solving a class of structured low rank approximations under the convex and unitarily invariant norm. By the aid of the variational inequality and monotone operator, the proposed method is proved to converge to the solution point of an equivalent variational inequality with a worst-case $\mathcal{O}(1/t)$ convergence rate in a nonergodic sense. We also analyze that the involved subproblems under the Frobenius norm are respectively equivalent to the structured least-squares problem and low rank least-squares problem, where the explicit solutions to some special cases are derived. In order to investigate the efficiency of the proposed method, several examples in system identification are tested to validate that the proposed method can outperform some state-of-the-art methods.

Keywords: Alternating update method; Low rank approximation; Structured matrix; System identification

AMS subject classifications. 41A29; 65F30; 65K10; 93B30

1 Introduction

Thought this paper, let $\mathbb{R}^{m\times n}$ be the set of $m\times n$ real matrices and $\mathbb{V}^{n\times n}$ be the set of all unitary matrices in $\mathbb{R}^{n\times n}$. The notations $\text{rank}(Y)$ and y_{ij} denote the rank and ij-th entry of matrix Y, respectively. The symbols $\mathbf{0}$ and $\mathbf{I}$ stand for the zero matrix and identity matrix with proper dimensions, respectively. Denoted by $\|\cdot\|_P$ be some convex and unitarily invariant norms. Then, by the following two well-known definitions, it is easy to verify that the Frobenius norm $\|\cdot\|_F$, the Spectral norm $\|\cdot\|_2$(the maximum of its singular values) and the Nuclear norm $\|\cdot\|_*$(the sum of its singular values) are convex and unitarily invariant, because any unitary transformation can not change the singular values of a matrix.

Definition 1.1 *A matrix norm $\|\cdot\|$ is unitarily invariant in $\mathbb{R}^{m\times n}$ if $\|QXN\| = \|X\|$ for any $Q\in\mathbb{V}^{m\times m}, N\in\mathbb{V}^{n\times n}, X\in\mathbb{R}^{m\times n}$.*

Definition 1.2 *A matrix norm $\|\cdot\|$ is convex in $\mathbb{R}^{m\times n}$ if*

$$\|\lambda X + (1-\lambda)Y\| \le \lambda\|X\| + (1-\lambda)\|Y\|$$

*The work was supported by the National Natural Science Foundation of China(11671318) and the Natural Science Foundation of Fujian Province(2016J01028).

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