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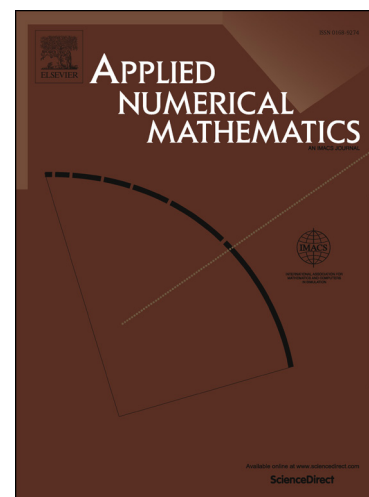
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Error Analysis of a Compact Finite Difference Method for Fourth-order Nonlinear Elliptic Boundary Value Problems*

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Abstract. This paper is concerned with a compact finite difference method with non-isotropic mesh sizes for a two-dimensional fourth-order nonlinear elliptic boundary value problem. By the discrete energy analysis, the optimal error estimates in the discrete L^2 , H^1 and L^∞ norms are obtained without any constraint on the mesh sizes. The error estimates show that the compact finite difference method converges with the convergence rate of fourth-order. Based on a high-order approximation of the solution, a Richardson extrapolation algorithm is developed to make the final computed solution sixth-order accurate. Numerical results demonstrate the high-order accuracy of the compact finite difference method and its extrapolation algorithm in the discrete L^2 , H^1 and L^∞ norms.

Keywords. Fourth-order nonlinear elliptic boundary value problem, compact finite difference method, error estimate, Richardson extrapolation.

1. Introduction

Boundary value problems of fourth-order nonlinear differential equations have been paid considerable attention in the research literature. Most of the discussions in this field were devoted to two-point boundary value problems, which arise from the static deflection of an elastic bending beam ([9,27]), the steady state of a prototype problem for the phase transition in condensed matter systems ([8,28]), and the study of travelling waves in a suspension bridge ([10,16]). In recent years, much attention has also been given to certain fourth-order elliptic boundary value problems in multiple dimensions (see [6,15,17–20,26,29,30]). In this paper, we focus on the following two-dimensional fourth-order nonlinear elliptic boundary value problem:

$$\begin{cases} \Delta(k(x, y)\Delta u) = f(x, y, u, \Delta u), & (x, y) \in \Omega, \\ u(x, y) = g(x, y), \quad \Delta u(x, y) = g^*(x, y), & (x, y) \in \partial\Omega, \end{cases} \quad (1.1)$$

where Ω is a rectangular domain or a union of rectangular domains, Δ is the Laplacian operator, $k(x, y)$ is a strictly positive C^2 -function on $\bar{\Omega} \equiv \Omega \cup \partial\Omega$, and the functions $f(x, y, u, v)$, $g(x, y)$ and $g^*(x, y)$ are regular enough in their respective domains. A physical interpretation of the above problem is that it governs the static deflection of a plate under a lateral loading. In this

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