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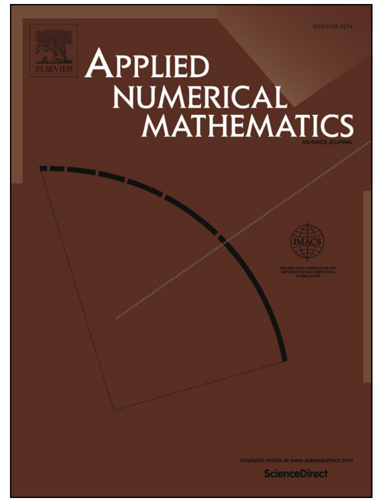


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# Error Analysis of a Compact Finite Difference Method for Fourth-order Nonlinear Elliptic Boundary Value Problems*

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**Abstract.** This paper is concerned with a compact finite difference method with non-isotropic mesh sizes for a two-dimensional fourth-order nonlinear elliptic boundary value problem. By the discrete energy analysis, the optimal error estimates in the discrete $L^2$, $H^1$ and $L^\infty$ norms are obtained without any constraint on the mesh sizes. The error estimates show that the compact finite difference method converges with the convergence rate of fourth-order. Based on a high-order approximation of the solution, a Richardson extrapolation algorithm is developed to make the final computed solution sixth-order accurate. Numerical results demonstrate the high-order accuracy of the compact finite difference method and its extrapolation algorithm in the discrete $L^2$, $H^1$ and $L^\infty$ norms.

**Keywords.** Fourth-order nonlinear elliptic boundary value problem, compact finite difference method, error estimate, Richardson extrapolation.

## 1. Introduction

Boundary value problems of fourth-order nonlinear differential equations have been paid considerable attention in the research literature. Most of the discussions in this field were devoted to two-point boundary value problems, which arise from the static deflection of an elastic bending beam ([9,27]), the steady state of a prototype problem for the phase transition in condensed matter systems ([8,28]), and the study of travelling waves in a suspension bridge ([10,16]). In recent years, much attention has also been given to certain fourth-order elliptic boundary value problems in multiple dimensions (see [6,15,17–20,26,29,30]). In this paper, we focus on the following two-dimensional fourth-order nonlinear elliptic boundary value problem:

$$\begin{cases} \Delta(k(x,y)\Delta u) = f(x,y,u,\Delta u), & (x,y) \in \Omega, \\ u(x,y) = g(x,y), \quad \Delta u(x,y) = g^*(x,y), & (x,y) \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega$ is a rectangular domain or a union of rectangular domains, $\Delta$ is the Laplacian operator, $k(x,y)$ is a strictly positive $C^2$-function on $\overline{\Omega} \equiv \Omega \cup \partial\Omega$, and the functions $f(x,y,u,v)$, $g(x,y)$ and $g^*(x,y)$ are regular enough in their respective domains. A physical interpretation of the above problem is that it governs the static deflection of a plate under a lateral loading. In this

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