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Single measurement experimental data for an inverse medium problem inverted by a multi-frequency globally convergent numerical method

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ABSTRACT

The recently developed globally convergent numerical method for an inverse medium problem with the data resulting from a single measurement, proposed in [23], is tested on experimental data. The data were originally collected in the time domain, whereas the method works in the frequency domain with the multi-frequency data. Due to a significant amount of noise in the measured data, a straightforward application of the Fourier transform to these data does not work. Hence, we develop a heuristic data preprocessing procedure, which is described in the paper. The preprocessed data are used as the input for the inversion algorithm. Numerical results demonstrate a good accuracy of the reconstruction of both refractive indices and locations of targets.

1. Introduction

In this work we consider an inverse medium problem for the Helmholtz equation in the three dimensional space $\mathbb{R}^3$. The objective is to reconstruct the coefficient of the Helmholtz equation in a bounded domain. The coefficient represents the spatially distributed dielectric constant of the medium. Our target application is in the detection and identification of explosives, such as antipersonnel mines and improvised explosive devices (IEDs). We calculate dielectric constants and estimate locations of objects which mimic explosives. Motivated by our target application, we use in our experiments only a single location of the source and, therefore, we use only a single resulting boundary measurement of the backscattered wave. Thus, this is the case of a single measurement data, which is one of the most challenging cases for any inversion algorithm.

The radar community relies mainly on the intensity of the radar images, which are obtained by migration-type imaging methods, to obtain geometrical information such as the shapes, the sizes, and the locations of the targets, see, e.g., [24,32,13]. Hence, the additional information about values of dielectric constants of targets of interest might help in the future to develop classification algorithms, which would better differentiate between explosives and clutter [24]. The targets in our experiments are located in air. It is known that, for example IEDs can be located in air. On the other hand, the case of targets buried in the ground is one of goals of our future research. We also note that this case was studied in [34]

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for experimental time dependent data using the previously developed globally convergent inverse algorithm of [6,7]. The algorithm of [6,7] works with the Laplace transform with respect to time of the solution to the hyperbolic equation (7), whereas the Helmholtz equation (2) we work with is the Fourier transform of the solution to that equation. In fact, the algorithm of [6,7] is similar with the one of both [23] and this paper; formulations of global convergence results are also similar.

Another term for the inverse medium problem under consideration is Coefficient Inverse Problem (CIP). Recently a globally convergent numerical method for this CIP with multi-frequency data resulting from a single measurement event has been developed by this group in [23]. This method was tested on computationally simulated data in [23] and on experimental multi-frequency data in [29]. The reason why we test this method on time dependent data is that testing of an inversion algorithm on several types of experimental data is a good idea, since it provides better assurances of the performance of this algorithm. Thus, the goal of the current paper is to test the technique of [23] on time dependent experimental data. The data of this paper were collected on a microwave scattering facility in the University of North Carolina at Charlotte. They were used in [8,33] to test a different globally convergent inverse algorithm of [6,7], which works with time dependent data.

It seems to be at the first glance that the easiest way to apply the frequency domain globally convergent algorithm of [23] to the time domain data is to apply the Fourier transform to the data and then to use the resulting data as the input for the algorithm. However, the straightforward application of this idea does not work here. The latter is due to a significant amount of noise in the measured data, see section 4.2.1. Such noise was observed earlier for both the time dependent [34,6,8,33] and multi-frequency [29] experimental data. We remark here that conventional data denoising techniques do not work for our data because of its complicated structure, see section 4.2.1. Therefore, it was concluded in [24,34,29,8,33] that a heuristic data preprocessing procedure is necessary to reduce the noise in the measured data. The data preprocessing procedure of the current paper is described in section 4. The result of this procedure is used as the input for the algorithm of [23].

The first step of the globally convergent numerical method of [34,6–8,33] is the application of the Laplace transform with respect to time to the solution of a hyperbolic wave-like PDE. However, it was observed in [29] that this technique does not work for the multi-frequency experimental data of [29]. The reason of this is that these data are stable only on a small interval of frequencies concentrated around a certain "optimal" frequency. A similar observation is made in section 4.2.7 for the Fourier transform of our preprocessed time dependent data. We remark that this phenomenon is not observed in computationally simulated data. In other words, when working with our data, one can rely only on a small interval of frequencies. We note that the latter is one of conditions of the global convergence theorem of [23]. The integral of the inverse Fourier transform, which is carried out over only that small interval of frequencies, cannot provide a reasonable accuracy of resulting time dependent data, and, therefore, it cannot provide a reasonable accuracy for the subsequent Laplace transform of the latter data.

The numerical methods of [6,7] and [23] are the so-called "approximately globally convergent methods". A detailed discussion of the notion of the approximate global convergence can be found in [6,7]. We now explain this notion briefly. Our CIP is a highly nonlinear one. For those CIPs which are highly nonlinear problems, an important question in its numerical treatment is: *How to obtain at least one point in a sufficiently small neighborhood of the exact solution without any advanced knowledge of this neighborhood*? A numerical method for a CIP is called "approximately globally convergent" (globally convergent in short, or GCM) if a theorem is proved, which claims that, under a certain reasonable mathematical assumption, this method addresses the above question positively, i.e. it delivers that point. We call this theorem the "global convergence theorem". The estimate of the distance between that point and the true coefficient should depend on the error in the data and some parameters of the discretization. We point out that the proximity of that point to the true coefficient is the main advantage of the GCM. Indeed, as soon as such a point is found, the solution can be refined via a small perturbation approach, see, e.g. Chapters 4 and 5 of [6]. That notion of a reasonable mathematical assumption is well justified by the well known fact that the goal of the development of such numerical methods for CIPs, which would positively address the above question, is a tremendously challenging one, especially for the case of a single measurement data. We refer to [6,7] for detailed discussions of the notion of the approximate global convergence.

We note that there is a vast literature on reconstruction methods developed for solving CIPs. To study the CIP under consideration, in which weak scattering assumptions are not applicable, the probably best known approach is nonlinear optimization, see, e.g., [12,15] and references therein. However, it is well-known that the methods based on nonlinear optimization schemes heavily rely on a strong *a priori* knowledge about the target. In particular, the convergence of those methods requires a good *a priori* initial approximation of the exact solution, that is, the starting point of iterations should be chosen to be sufficiently close to the solution. Hence, we call such methods "locally convergent". Note that in our desired applications such *a priori* knowledge is not always available. Concerning qualitative reconstruction methods for inverse scattering problems, we refer to [4,3,11,20,17,26–28] and the references therein. These methods do not require good first guesses. However, they reconstruct only the shapes of scattering objects instead of their material properties.

Finally, we refer to some different globally convergent numerical methods for solving CIPs with multiple measurements of the Dirichlet-to-Neumann map [1,2,19,18,30]. These techniques were tested on computationally simulated data in [19,18]. We recall that our GCM in this paper deals with only a single backscatter measurement.

The paper is organized as follows. We present in section 2 the formulation of the direct and inverse problems considered in this paper. Section 3 is a brief summary of our GCM. The data collection and preprocessing are described in section 4.