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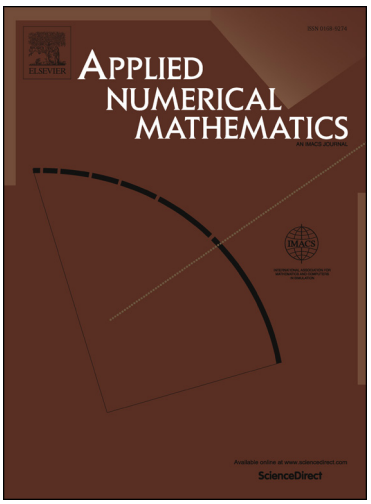

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Exponentially graded mesh for a singularly perturbed problem with two small parameters

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Abstract

A one–dimensional singularly perturbed boundary value problem with two small perturbation parameters is numerically solved on an exponentially graded mesh. Using an $h-$version of the standard Galerkin method with higher order polynomials, we prove a robust convergence in the corresponding energy norm. Numerical experiments support theoretical findings.

Keywords: singularly perturbed problem, two small parameters, Galerkin finite element method, exponentially graded mesh

2010 MSC: 65L11, 65L60, 65L70

1. Introduction

We consider the following singularly perturbed boundary value problem

$$\begin{cases} -\varepsilon_1 u''(x) + \varepsilon_2 b(x)u'(x) + c(x)u(x) = f(x), \quad x \in \Omega := (0,1), \\ u(0) = 0, \quad u(1) = 0, \end{cases} \tag{1}$$

with two small perturbation parameters $0 < \varepsilon_1, \varepsilon_2 \ll 1$. Let the data functions b, c, f be sufficiently smooth on $\bar{\Omega} = [0,1]$ and

$$b(x) \geq \beta > 0, \quad c(x) \geq \gamma > 0, \quad c(x) - \frac{\varepsilon_2}{2} b'(x) \geq \varrho > 0, \quad x \in \Omega, \tag{2}$$

where β, γ, ϱ are constants. There are two limiting cases for the perturbation parameter ε_2:

- for $\varepsilon_2 = 0$, the problem (1)–(2) is the well–known reaction–diffusion problem whose solution has two boundary layers of the widths $\mathcal{O}(\varepsilon_1^{1/2} |\ln \varepsilon_1^{1/2}|)$;

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