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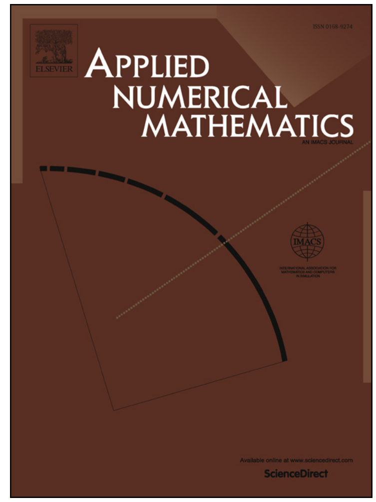

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Finite element method for a symmetric tempered fractional diffusion equation

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Abstract

A space fractional diffusion equation involving symmetric tempered fractional derivative of order $1 < \alpha < 2$ is considered. A Galerkin finite element method is implemented to obtain spatial semi-discrete scheme and first order centered difference in time is used to find a fully discrete scheme for tempered fractional diffusion equation. We construct a variational formulation and show its existence, uniqueness and regularity. Stability and error estimates of numerical scheme are discussed. The theoretical and computational study of accuracy and consistence of the numerical solutions are presented.

Keywords: Tempered fractional derivative, Galerkin finite element method, Crank-Nicolson method.

1 Introduction

A fractional diffusion equation with space tempered fractional derivative is proposed by Cartea and del Castillo-Negrete [1] which governs the random walk with an exponentially truncated power law jump distribution introduced by Mantegna and Stanley [2] and Koponen [3]. Another diffusion model in heterogeneous media with time tempered fractional derivative operator by selecting a specific memory function is suggested by Meerschaert et al. [4]. Approximations to the fractional differential equations with tempered fractional derivatives which generally occur in physics, biology and finance, are studied by many authors. Baeumer and Meerschaert [5] presented particle tracking methods and finite difference method to solve the tempered fractional equation with drift. Sabzikar et al. [6] developed a difference formulation to approximate the fractional tempered derivative. Hanert and Piret [7] used the pseudospectral method based on Chebyshev polynomial expansion for solving space and time tempered fractional diffusion equation. Zhang et al. [8] developed Lagrangian scheme to solve a time tempered anomalous diffusion model in multidimensional heterogeneous media. Li and Deng [9] proposed a high order difference formulation to approximate the tempered fractional diffusion equation for initial and nonhomogeneous Dirichlet boundary value problem. Zhang et al. [10] applied finite difference scheme with second order accuracy to solve the tempered fractional Black-Scholes equation. As far as we know, finite element method has not been considered for a tempered fractional diffusion equation. Therefore, we investigate a Galerkin finite element method for this problem.

In Section 2, we define an initial and boundary value problem for the diffusion equation with symmetric space tempered fractional operator. We recall preliminaries on tempered calculus and study tempered fractional derivatives as continuous operators on fractional Sobolev spaces. In Section 3, we write a variational formulation including symmetric, coercive and continuous bilinear form with the aid of given transformation. In Section 4, we prove the existence, uniqueness and regularity of the variational solution associated with initial and source functions. In Section 5, we give error estimates and stability of fully discrete scheme implementing a Galerkin finite element method with Crank-Nicolson discretization. In the last section, we give a numerical experiment to illustrate the accuracy of the scheme.