Applied Numerical Mathematics 117 (2017) 115–138

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

Two-dimensional Shannon wavelet inverse Fourier technique for pricing European options

G. Colldeforns-Papiol [a,b,*], L. Ortiz-Gracia [c], C.W. Oosterlee [d,e]

[a] Centre de Recerca Matemàtica, Campus de Bellaterra, Edifici C, 08193 Bellaterra (Barcelona), Spain
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain
[c] Department of Econometrics, University of Barcelona, Diagonal 690, 08034 Barcelona, Spain
[d] CWI – Centrum Wiskunde & Informatica, NL-1090 GB Amsterdam, The Netherlands
[e] Delft University of Technology, Delft Institute of Applied Mathematics, 2628 CD Delft, The Netherlands

ARTICLE INFO

Article history:
Received 3 June 2016
Received in revised form 7 January 2017
Accepted 9 March 2017
Available online 16 March 2017

Keywords:
Option pricing
European options
Two-colour rainbow options
Basket options
Spread options
Lévy process
Shannon wavelets
Cardinal sine function
Fourier transform inversion

ABSTRACT

The SWIFT method for pricing European-style options on one underlying asset was recently published and presented as an accurate, robust and highly efficient technique. The purpose of this paper is to extend the method to higher dimensions by pricing exotic option contracts, called rainbow options, whose payoff depends on multiple assets. The multidimensional extension inherits the properties of the one-dimensional method, being the exponential convergence one of them. Thanks to the nature of local Shannon wavelets basis, we do not need to rely on a-priori truncation of the integration range, we have an error bound estimate and we use fast Fourier transform (FFT) algorithms to speed up computations. We test the method for similar examples with state-of-the-art methods found in the literature, and we compare our results with analytical expressions when available.

1. Introduction

Financial derivatives such as options are traded all over the world. In the general class of exotic options that are not listed on regulated exchanges, multi-asset options form a class for which efficient solution methods are not easily obtained.

Analytical formulae to price multi-asset derivatives are only available for the most simple cases. Hence, there is a need to develop numerical methods to approximate their prices and develop efficient algorithms to implement them, so that they provide useful information in a market that changes rapidly. Recently, multidimensional option pricing has become an important topic, but this is an area with high computational demands. Some examples of multidimensional options are exotic option contracts called multicolour rainbow options whose payoff depends on multiple assets.

One of the most commonly used methods for pricing options is Monte Carlo simulation. This method has the advantage of scaling linearly with the number of dimensions. However, convergence is slow and a large number of simulations is needed if accurate results are desired. Different approaches are based on partial differential equations (PDE) and Fourier

* Corresponding author.
E-mail addresses: gcolldeforns@crm.cat, gemma.colldeforns@gmail.com (G. Colldeforns-Papiol), luis.ortiz-gracia@ub.edu (L. Ortiz-Gracia), c.w.oosterlee@cwi.nl (C.W. Oosterlee).

http://dx.doi.org/10.1016/j.apnum.2017.03.002
0168-9274/

methods. This last class of approximations relies on a transformation to the Fourier domain. The probability density function appears in the time domain, and it is not known for many relevant multidimensional processes. However, its characteristic function, this is its Fourier transform, is often available in closed form. Nevertheless, in both PDE and Fourier-based methods the curse of dimensionality plays a prominent role. The curse of dimensionality is the exponential growth of the complexity of the problem when the dimension increases, and modern computer systems cannot handle this huge amount of computations. For this reason, and despite their drawbacks, Monte Carlo methods are the most commonly used alternatives when the dimension is bigger than four or five, depending on the specific product.

One well-developed multidimensional Fourier-based method is the multidimensional COS method presented in [11] and called 2D-COS when the dimension is two. However, it may exhibit problems in the vicinity of the integration boundaries because of the periodic behaviour of cosines. This problem becomes evident for long maturity options, where round-off errors may accumulate near domain boundaries. Also for short maturity options, typically governed by a highly peaked density function, many cosine terms may be needed for an accurate representation. In addition, an accurate integration interval is important to capture the whole mass of the recovered density, but the choice of the interval is entirely based on the cumulants, which sometimes are not easily available or do not provide a good truncation range.

In the one-dimensional case, local wavelets bases have been considered in [9,10], which overcome some of the problems of the well known one-dimensional COS method [2]. Wavelets give flexibility and enhance robustness when pricing long maturity options and heavy tailed asset processes. In particular, Shannon wavelets used in the SWIFT method [10], are smooth wavelets based on the cardinal sine (sinc in short) function. The SWIFT method has been used for European option pricing and it is based on a wavelet expansion of the underlying density function recovered from its Fourier transform. Haar wavelets and B-splines of order one were used in [9]. These wavelets have compact support and the pricing formula is particularly easy to implement with the Haar basis, although the method converges slower than SWIFT. Higher order B-splines are considered in [3].

The aim of the present work is to extend the one-dimensional SWIFT method to a higher dimension to be able to price European-style financial contracts with a payoff depending on more than one asset. For two-dimensional contracts we call it 2D-SWIFT. The unknown density function is approximated in terms of a finite combination of multidimensional Shannon scaling functions and the coefficients of the approximation are recovered by inverting its Fourier transform. Then, the payoff coefficients are calculated by means of the approximated density within the discounted expected payoff pricing formula setting, and the final price of the contract is readily obtained. Central to this two-step process is the use of an appropriate approximation of the sinc function. As opposed to the one-dimensional case, where the Vieta's formula was employed, we approximate the sinc function in terms of a sum of complex exponentials. Proceeding this way, the algebraic manipulation to obtain the pricing formulae is drastically simplified. Further, we provide an error analysis which facilitates the choice of the parameters of the new method and enhance the overall speed with an FFT algorithm. We test our method with examples and methods from the literature, like for instance the well-known 2D-COS in [11]. We price basket (both geometric and arithmetic), spread, call-on-max, put-on-min, and correlation options. It is worth mentioning that we can perform a consistency check of our method, since there exists a closed form solution for spread options with strike zero (the Margrabe formula [6]), as well as an analytical method for the valuation of a geometric basket option (since it can be transformed into a one-dimensional European option). The 2D-SWIFT method appears to be a very competitive pricing machinery, showing exponential convergence with a very short run time. We also benefit from the local behaviour of Shannon wavelets. Due to that fact, the accurate treatment of options with long maturities is possible since we can remove the density coefficients affected by the exponential growth of the payoff without changing the remaining part of the approximation. Finally, it is not necessary to rely on a-priori truncation of the integration range. We use an initial guess of the truncation range, which allows to compute the density coefficients much faster with an FFT algorithm, and we adaptively compute the final integration range if necessary. It is worth remarking that the number of terms needed in the expansion is automatically calculated once the scale of approximation has been fixed, which is a central step in our method.

The outline of this paper is as follows. We start in Section 2 with the presentation of the two-dimensional option pricing problem and the related theory of the multidimensional wavelets framework. In Section 3 the 2D-SWIFT pricing formula is derived and the respective extension to higher dimensions is also introduced. In Section 4 we present an error analysis along with a study on how to select the parameters of the method. Numerical tests are performed in Section 5 and a specific study of 2D-SWIFT strengths is carried out in Section 6, such as the behaviour for extreme maturities or the automatic computation of the number of coefficients. Section 7 concludes.

2. Motivation: rainbow option pricing

In this section we define the two-dimensional pricing formula as a discounted expectation of the option value at expiration. From now on, bold letters will denote vectors.

Assume $(\Omega, \mathcal{F}, P)$ is a probability space, $T > 0$ is the finite terminal time, and $\mathbb{F} = (\mathcal{F}_S)_{0\leq S\leq T}$ is a filtration with the usual conditions. Then, the process $\mathbf{X}_t = (X_t^1, X_t^2)$ denotes a two-dimensional stochastic process on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$, representing the log-asset prices of the underlying. We assume that the bivariate characteristic function of the stochastic process is known.

The value of a European rainbow option with payoff function $g(\cdot)$, which depends on the underlying asset price, is given by the risk-neutral option valuation formula,