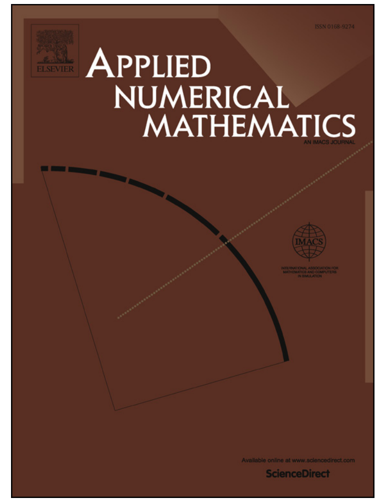

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High-order numerical solution of the Helmholtz equation for domains with reentrant corners[☆]

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Abstract

Standard numerical methods often fail to solve the Helmholtz equation accurately near reentrant corners, since the solution may become singular. The singularity has an inhomogeneous contribution from the boundary data near the corner and a homogeneous contribution that is determined by boundary conditions far from the corner. We present a regularization algorithm that uses a combination of analytical and numerical tools to distinguish between these two contributions and ultimately subtract the singularity. We then employ the method of difference potentials to numerically solve the regularized problem with high-order accuracy over a domain with a curvilinear boundary. Our numerical experiments show that the regularization successfully restores the design rate of convergence.

Keywords: singularity subtraction, regularization, asymptotic expansion near singularity, difference potentials, curvilinear boundaries, compact differencing,

1. Introduction

We consider a time-harmonic wave problem on a bounded 2D domain with a reentrant corner, as shown schematically in Figure 1. Problems with reentrant corners are difficult because the solution may become singular near the corner, i.e., the derivatives of the solution become unbounded. Standard numerical methods perform poorly near singularities, so they must be modified before use on singular problems. Wave problems with reentrant corners may arise, for instance, when analyzing the scattering of radar waves near an air–ocean–sea ice interface [1]. Marin et al. have solved several Helmholtz-type equations on domains with reentrant corners in [2] and [3] with the boundary element method (BEM) and the method of fundamental solutions (MFS), respectively. Martinsson [4] has applied an efficient spectral method to the Helmholtz equation on an L-shaped domain, though the method cannot handle arbitrarily-shaped boundaries and there is no attempt to remove the singularity. A variety of techniques [5], such as the Method of Particular Solutions (MPS) [6, 7], have been applied to the eigenvalue problem for the Laplacian on an L-shaped domain.

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