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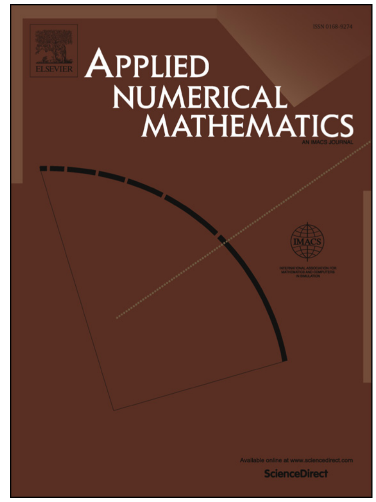

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Galerkin finite element method for the nonlinear fractional Ginzburg-Landau equation ⋆

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Abstract

In this paper, we are concerned with the numerical solution of the nonlinear fractional Ginzburg-Landau equation. Galerkin finite element method is used for the spatial discretization, and an implicit midpoint difference method is employed for the temporal discretization. The boundedness, existence and uniqueness of the numerical solution, and the unconditional error estimates in the L^2-norm are investigated in details. To numerically solve the nonlinear system, linearized iterative algorithms are also considered. Finally, some numerical examples are presented to illustrate the effectiveness of the algorithm.

Key words: Fractional Ginzburg-Landau equation, Fractional Laplacian, Riesz fractional derivative, Implicit midpoint scheme, Finite element method, Convergence

1. Introduction

The fractional Ginzburg-Landau equation (FGLE), considered as a generalization of the classical (integer order) one, has been proposed to describe a vast variety of nonlinear phenomena. For example, Tarasov et al. [1, 2] derived the fractional generalization of Ginzburg-Landau equation from the variational Euler-Lagrange equation for fractal media. Since the fractals can be realized in nature as a fractal process or fractal media, there are wide range of applications for the FGLE in physical phenomena, such as the dynamical processes in a medium with fractal dispersion [1], a fairly general class of critical phenomena when the organization of the system near the phase transition point is influenced by a competing nonlocal ordering [3] and a network of diffusively Hindmarsh-Rose (HR) neurons with long-range synaptic coupling [4].

Theoretical analysis for the FGLE has been well done [5–9]. Tarasov [5] presented the psi-series for the one-dimensional FGLE and obtained the leading-order behaviours of solutions about

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