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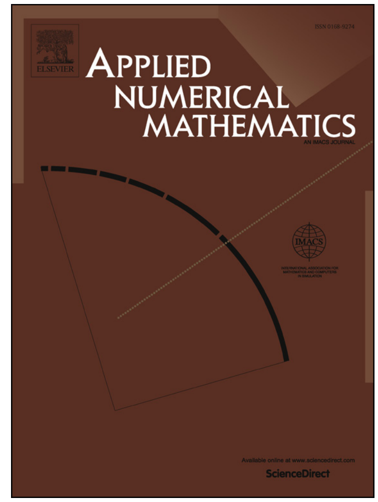

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Analysis of output-based error estimation for finite element methods

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Abstract

In this paper, we develop *a priori* estimates for the convergence of outputs, output error estimates, and localizations of output error estimates for Galerkin finite element methods. Output error estimates for order p finite element solutions are constructed using the Dual-Weighted Residual (DWR) method with a higher-order $p' > p$ dual solution. Specifically, we analyze these DWR estimates for Continuous Galerkin (CG), Discontinuous Galerkin (DG), and Hybridized DG (HDG) methods applied to the Poisson problem. For all discretizations, as $h \rightarrow 0$, we prove that the output and output error estimate converge at order $2p$ and $2p'$ (assuming sufficient smoothness), while localizations of the output and output error estimate converge at $2p+d$ and $p+p'+d$. For DG, the results use a new post processing for the error associated with the lifting operator. For HDG, these rates improve an additional order when the stabilization is based upon an $O(1)$ length scale.

Keywords: Output functional estimation; Finite Element Method; Discontinuous Galerkin; Hybridizable Discontinuous Galerkin

1. Introduction

A posteriori estimation aims to provide computable estimates of numerical error in the discrete approximate solutions of Partial Differential Equations (PDEs). The computed *a posteriori* estimate can then be used to correct the output and/or drive mesh adaptation. Energy-based *a posteriori* error estimates have

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