

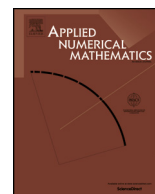


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Partition of unity interpolation using stable kernel-based techniques

R. Cavoretto^{a,*}, S. De Marchi^b, A. De Rossi^a, E. Perracchione^a, G. Santin^c

^a Department of Mathematics "G. Peano", University of Torino, via Carlo Alberto 10, I-10123 Torino, Italy

^b Department of Mathematics, University of Padova, via Trieste 63, I-35121 Padova, Italy

^c Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany

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This paper is dedicated to Prof. Francesco A. Costabile on the occasion of his 70th birthday

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ABSTRACT

In this paper we propose a new stable and accurate approximation technique which is extremely effective for interpolating large scattered data sets. The Partition of Unity (PU) method is performed considering Radial Basis Functions (RBFs) as local approximants and using locally supported weights. In particular, the approach consists in computing, for each PU subdomain, a stable basis. Such technique, taking advantage of the local scheme, leads to a significant benefit in terms of stability, especially for flat kernels. Furthermore, an optimized searching procedure is applied to build the local stable bases, thus rendering the method more efficient.

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1. Introduction

Considering the state of the art [5,6,13,14,24], we propose a new method for multivariate approximation which allows to interpolate large scattered data sets stably, accurately and with a relatively low computational cost.

The interpolant we consider is expressed as a linear combination of some basis or kernel functions. Focusing on Radial Basis Functions (RBFs), the Partition of Unity (PU) is performed by blending RBFs as local approximants and using locally supported weight functions. With this approach a large problem is decomposed into many small problems, [2,17,23,30], and therefore in the approximation process we could work with a large number of nodes.

However, in some cases, local approximants and consequently also the global one suffer from instability due to ill-conditioning of the interpolation matrices. This is directly connected to the order of smoothness of the basis function and to the node distribution. It is well-known that the stability depends on the flatness of the RBF. More specifically, if one keeps the number of nodes fixed and considers smooth basis functions, then the problem of instability becomes evident for small values of the shape parameter. Of course, a basis function with a finite order of smoothness can be used to improve the conditioning but the accuracy of the fit gets worse. For this reason, the recent research is moved to the study of more stable bases.

* Corresponding author.

E-mail addresses: roberto.cavoretto@unito.it (R. Cavoretto), demarchi@math.unipd.it (S. De Marchi), alessandra.derossi@unito.it (A. De Rossi), emma.perracchione@unito.it (E. Perracchione), santinge@mathematik.uni-stuttgart.de (G. Santin).

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Table 1

Examples of strictly positive definite radial kernels with their orders of smoothness and shape parameter $\varepsilon > 0$; $r = \|\cdot\|_2$ is the Euclidean distance, while $(\cdot)_+$ denotes the truncated power function.

RBF	$\phi_\varepsilon(r)$
Gaussian C^∞ (GA)	$e^{-\varepsilon^2 r^2}$
Inverse MultiQuadric C^∞ (IMQ)	$(1 + \varepsilon^2 r^2)^{-1/2}$
Matérn C^6 (M6)	$e^{-\varepsilon r} (\varepsilon^3 r^3 + 6\varepsilon^2 r^2 + 15\varepsilon r + 15)$
Matérn C^4 (M4)	$e^{-\varepsilon r} (\varepsilon^2 r^2 + 3\varepsilon r + 3)$
Wendland C^6 (W6)	$(1 - \varepsilon r)_+^8 (32\varepsilon^3 r^3 + 25\varepsilon^2 r^2 + 8\varepsilon r + 1)$
Wendland C^4 (W4)	$(1 - \varepsilon r)_+^6 (35\varepsilon^2 r^2 + 18\varepsilon r + 3)$

For particular RBFs, techniques allowing to stably and accurately compute the interpolant, also in the *flat limit* $\varepsilon \rightarrow 0$, have been designed in the recent years. These algorithms, named RBF-QR methods, are all rooted in a particular decomposition of the kernel, and they have been developed so far to treat the Gaussian and the Inverse MultiQuadric kernel. We refer to [18,20–22] for further details on these methods.

A different and more general approach, consisting in computing, via a truncated Singular Value Decomposition (SVD) stable bases, namely Weighted SVD (WSVD) bases, has been presented in [13]. We remark that in the cases where the RBF-QR algorithms can be applied, they produce a far more stable solution of the interpolation problem. Nevertheless, the present technique applies to *any* RBF kernel, and to any domain.

In this paper, a stable approach via the PU method, named WSVD-PU, which makes use of local WSVD bases and uses compactly supported weight functions, is presented. Thus, following [14], for each PU subdomain a stable RBF basis is computed in order to solve the local interpolation problem. Consequently, since the local approximation order is preserved for the global fit, the interpolant results more stable and accurate. Concerning the stability, we surely expect a more significant improvement in the stabilization process with infinitely smooth functions than with functions characterized by a finite order of regularity. Moreover, in terms of accuracy, the benefits coming from the use of such stable bases are more significant in a local approach than in a global one. In fact, generally, while in the global case a large number of truncated terms of the SVD must be dropped to preserve stability, a local technique requires only few terms are eliminated, thus enabling the method to be much more accurate.

Concerning the computational complexity of the algorithm, the use of the so-called block-based space partitioning data structure enables us to efficiently organize points among the different subdomains, [7]. Then, for each subdomain a local RBF problem is solved with the use of a stable basis. The main and truly high cost, involved in this step, is the computation of the SVD. To avoid this drawback, techniques based on Krylov space methods are employed, since they turn out to be really effective, [3,14]. A complexity analysis supports our findings.

The guidelines of the paper are as follows. In Section 2, we present the WSVD bases, computed by means of the Lanczos algorithm, in the general context of global approximation. Such method is used coupled with the PU approach which makes use of an optimized searching procedure, as shown in Section 3. The proposed approach turns out to be stable and efficient, as stressed in Section 4. In Sections 5 and 6 extensive numerical experiments and applications, carried out with both globally and compactly supported RBFs of different orders of smoothness, support our results. Finally, Section 7 deals with concluding remarks. Moreover, all the MATLAB codes are made available to the scientific community in a downloadable free software package:

<http://hdl.handle.net/2318/1527447>.

2. RBF interpolation and WSVD basis

In Subsection 2.1 we briefly review the main theoretical aspects concerning RBF interpolation, [4], while the remaining subsections are devoted to the efficient computation of the WSVD basis via Krylov space methods.

2.1. RBF interpolation

Our goal is to recover a function $f : \Omega \rightarrow \mathbb{R}$, Ω being a bounded set in \mathbb{R}^M , using a set of samples of f on N pairwise distinct points $X_N \subset \Omega$, namely $\mathbf{f} = [f_1, \dots, f_N]^T$, $f_i = f(\mathbf{x}_i)$, $\mathbf{x}_i \in X_N$. To this end, one considers a positive definite and symmetric kernel $\Phi : \Omega \times \Omega \rightarrow \mathbb{R}$ to construct an interpolant in the form

$$R^N(\mathbf{x}) = \sum_{j=1}^N c_j \Phi(\mathbf{x}, \mathbf{x}_j), \quad \mathbf{x} \in \Omega. \quad (1)$$

The kernels we consider are always radial, meaning that there exist a positive *shape parameter* ε and a function $\phi : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ such that for all $\mathbf{x}, \mathbf{y} \in \Omega$, $\Phi(\mathbf{x}, \mathbf{y}) = \phi_\varepsilon(\|\mathbf{x} - \mathbf{y}\|_2) = \phi(\varepsilon\|\mathbf{x} - \mathbf{y}\|_2)$. In Table 1 we report a list of some strictly positive

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