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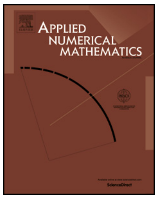

Performances and limitations of the diffusive approximation of the 2-d shallow water equations for flood simulation in urban and rural areas

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ABSTRACT

The Shallow Water Equations (SWE) are a time-dependent system of non-linear partial differential equations of hyperbolic type. Flood propagation in rivers and in the neighbouring areas is a typical example for which the use of the SWE is required. The numerical integration of the SWE needs complicated schemes that still require a significant computational effort. This fact has maintained interest in techniques that can approximate simulations from full two-dimensional (2-D) shallow water models with less computation heaviness. In particular, an approximation of the full SWE consists in neglecting the inertial terms, leading to a degradation of the original hyperbolic model to a parabolic one. This approximation is traditionally justified by the fact that flooding over plain areas is characterized by a slow evolution. Although there are important papers on model benchmarking and comparative analysis of flood propagation models, detailed analyses of effective ability and limitations of simplified models in reproducing floods processes are still rare, especially for urban environments. Therefore, this paper aims at providing a contribution to the model benchmarking and to the influence induced by the application of simplified models on the numerical simulations of flood events, overcoming some limitations that characterize part of the studies in the literature. In particular, on the one hand, more complicated experimental tests will be considered and, on the other hand, the same numerical grid for every model considered herein will be used in order to remove its effect from the numerical results. The simulations discussed here seem to suggest that the use of diffusive-type models is questionable, especially in urban districts, due to the poor predictions of the events that might be simulated around the buildings. Conversely, the application of the shallow water model gave excellent results in all the situations considered in this paper and, therefore, its use is recommended to obtained reliable estimations of flood hazard.

1. Introduction

A wide variety of physical phenomena of practical interest to many scientists and engineers involves water flows with the free surface under the influence of gravity. Among these, typical examples are tides in oceans, breaking waves on shallow beaches, surges and dam-break wave modelling and flooding. Regarding the last topic, the numerical modelling of flood

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inundation in urban areas is becoming a key tool for the accurate calculation of water depths and velocities, which are essential for the flood risk assessment [26,47]. The reason for this increasing interest lies in the unacceptable costs in terms of property damage and even human lives lost that usually occur when an urban area is affected by flooding.

It is generally accepted that the unsteady flow of water may be described by the shallow water equations (SWE). Until a few decades ago, the SWE were essentially developed following a one-dimensional schematization, using various devices for describing the propagation in the areas at the sides of the river. However, this schematization may suffer from inherent limitations, which do not always put us in a position to properly simulate the phenomena that take place in the riverside areas (see for example Costabile and Macchione [14]; Costabile et al. [17,18]). For this reason, today it is customary to use the SWE in a two-dimensional schematization, although the 1-D approach may remain competitive for calculating, more quickly, the propagation in the mountain reach of the river [39].

The SWE represent mass and momentum conservation and can be obtained by depth averaging the Navier–Stokes equations in the vertical direction, assuming that the wave length of the phenomenon, they are supposed to describe, has to be much higher than the depth of the water where the phenomenon takes place.

From a mathematical point of view the shallow water equations are a time-dependent system of nonlinear partial differential equations of hyperbolic type.

One of the most challenging aspects of the shallow water wave equations is due to their non-linear character, which yields both discontinuous and smooth solutions. In particular, discontinuous solutions in finite time can be obtained even considering smooth initial data. The non-linear character of the equations also means that only in very special cases is it possible to derive analytical solutions to these equations. Indeed, the SWE can rarely be exactly solved and even present many difficulties for numerical computation because of the discontinuities often called shock waves.

In order to overcome the discontinuities problem due to shock, many numerical schemes have been developed. In this context, widely used approaches to solve the SWE are the so-called Godunov-type Finite Volume Methods [57,58], which allow the computation of numerical fluxes.

In fluvial engineering applications, the presence of irregular topographies poses further numerical difficulties that are also being tackled in the literature in order to obtain stable and reliable results. In particular, attention has to be paid to the numerical treatment of the source terms of the SWE in order to avoid spurious oscillations due to a bad balance scheme for pressure and gravity forces. For this reason, the numerical integration of the source term should be carried out in agreement with the flux functions. Another numerical challenging problem concerns the management of small water depths near wet/dry interfaces that requires specific algorithms in order to avoid numerical instabilities.

While the efficacy of the SWE approach has been discussed several times, recently, not only in urban areas [22,12,17,44] but also for overland flow simulations [15,16,30,13], coastal and channels morphodynamics [9,35], tsunamis simulations [24], its numerical integration needs complicated schemes that still require a significant computational effort.

Although the reduction of the associated computational times can be achieved using parallel computations or GPU programming [59], several techniques that can approximate the solutions provided by the two-dimensional shallow water models with fewer computations were developed. Recent examples include the integration of 1-D and 2-D approaches [37,41], porosity-based methods for representing sub-grid scale features in coarse resolution models [28,60,48,27,34], models that consider inertia and diffusion but ignore advection [4,6,1,61,50,40], diffusive models [45,2,21,55,37].

In particular, an approximation of the full SWE consists in neglecting the inertial terms, leading to a degradation of the original hyperbolic model to a parabolic one. This approximation is traditionally justified by the fact that, except for some situations of practical interest, flooding over plain areas is characterized by a slow development. The diffusive wave simplification neglects the inertial terms allowing, therefore, a simplified set of equations to be solved. Moreover, as recalled by Aricò et al. [3], only one boundary condition is required at each boundary cell, where the appropriate number of boundary conditions in the fully dynamic case depends on the local Froude number.

In general terms using a simplified set of equations leads to faster computational times. However, due to stability criterion, some authors observed that diffusive models could be computationally less effective than dynamic models when high resolution meshes are used. For that reason, parallelized versions of a two-dimensional diffusive wave model with adaptive time step have been developed in the literature [36].

In the literature, there are several studies related to the application of 2-D numerical models based on the diffusive wave equations, mainly referred to inundations due to slow-varying floods (see for example Aronica et al. [5]; Bradbrook et al. [10]). Other studies have looked at benchmarking two-dimensional shallow water models [29], focusing also on urban settings [23]. Hunter et al. [31] compared three full shallow water codes and two diffusive codes for an urban test site in Glasgow, UK. They found some differences in the depth and extent dynamics, due to the different representations of the physical process and numerical solvers used. Néelz and Pender [43] benchmarked the majority of industry codes used for flood risk modelling in the UK in a number of numerical cases. Similarly, Neal et al. [42] benchmarked three two-dimensional explicit hydraulic models, which can be broadly defined as simulating diffusive, inertial or shallow water waves using test cases for which results from industry models are also available. They showed that for gradually varied flow, fully dynamic shallow water models may be unnecessarily complex, and simpler, cheaper schemes can perform just as well, both in terms of velocity and depths. However, in situations characterized by low friction and supercritical flow, simplified models might become unstable and mass balance errors become large.

Although there are important papers on model benchmarking and comparative analysis of flood propagation models, detailed analyses of effective ability and limitations of simplified models in reproducing floods processes are still rare, es-