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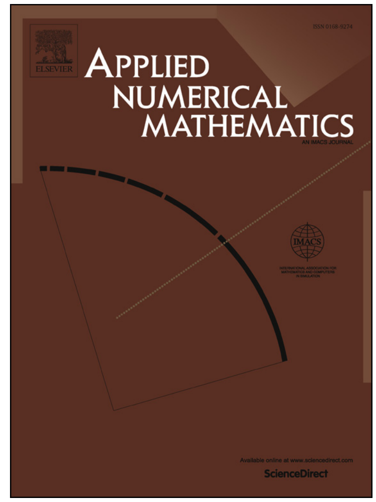

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A numerical method for solving the time variable fractional order mobile-immobile advection-dispersion model

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Abstract

In this article, we proposed a new numerical method to obtain the approximation solution for the time variable fractional order mobile-immobile advection-dispersion model based on reproducing kernel theory and collocation method. The equation is obtained from the standard advection-dispersion equation(ADE) by adding the Coimbra's variable fractional derivative in time of order $\gamma(x,t) \in [0,1]$. In order to solve this kind of equation, we discuss and derive the ε-approximate solution in the form of series with easily computable terms in the bivariate spline space. At the same time, the stability and convergence of the approximation is investigated. Finally, numerical examples are provided to show the accuracy and effectiveness.

Keywords: Mobile-immobile, Advection-dispersion equation, Spline space, Fractional derivative

1. Introduction

In this paper, we consider the following time variable fractional order mobile-immobile advection-dispersion model

$$\beta_1 \frac{\partial C(x,t)}{\partial t} + \beta_2 D_t^{\gamma(x,t)} C(x,t) = -V \frac{\partial C(x,t)}{\partial x} + D \frac{\partial^2 C(x,t)}{\partial x^2} + f(x,t) \tag{1}$$

subject to the initial and boundary conditions

$$\begin{aligned} &C(x,0) = \phi(x), \quad 0 < x < 1 \\ &C(0,t) = \varphi_1(t), \quad C(L,t) = \varphi_2(t), \quad 0 < t < 1 \end{aligned} \tag{2}$$

where $(x,t) \in \Omega = [0,1] \times [0,1], \beta_1 \geq 0, \beta_2 \geq 0, V > 0, D > 0, 0 < \gamma_{min} \leq \gamma(x,t) \leq \gamma_{max} < 1$, $\gamma(x,t), f(x,t) \in C(\Omega)$, and $D_t^{\gamma(x,t)}$ is the Coimbra variable order derivative operator [1].

The numerical solution of ADE has attracted the interest of many authors. The mobile-immobile model is an intention of ADE, it can describe the movement of solute transport in both porous and fractured media [2, 3]. Recent years, more and more researchers are finding that the advection-dispersion

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