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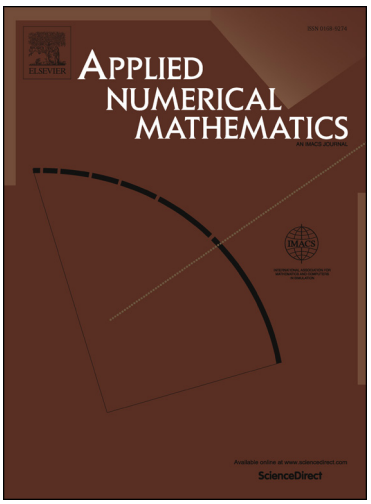

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Order reduction phenomenon for general linear methods

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Abstract

The order reduction phenomenon for general linear methods (GLMs) for stiff differential equations is investigated. It turns out that, similarly as for standard Runge-Kutta methods, the effective order of convergence for a large class of GLMs applied to stiff differential systems, is equal to the stage order of the method. In particular, it is demonstrated that the global error $\|e^{[n]}\|$ of GLMs of order p and stage order q applied to the Prothero-Robinson test problem $y'(t) = \lambda(y(t) - \varphi(t)) + \varphi'(t)$, $t \in [t_0, T]$, $y(t_0) = \varphi(t_0)$, is $\mathcal{O}(h^q) + \mathcal{O}(h^p)$ as $h \to 0$ and $h\lambda \to -\infty$. Moreover, for GLMs with Runge-Kutta stability which are $A(0)$-stable and for which the stability function $R(z)$ of the underlying Runge-Kutta methods, (i.e., the corresponding RK methods which have the same absolute stability properties as the GLMs), is such that $R(\infty) \neq 1$, the global error satisfies $\|e^{[n]}\| = \mathcal{O}(h^{q+1}) + \mathcal{O}(h^p)$ as $h \to 0$ and $h\lambda \to -\infty$. These results are confirmed by numerical experiments.

Keywords: General linear methods, order conditions, linear stability analysis, Prothero-Robinson problem, stiff differential systems, order reduction phenomenon

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