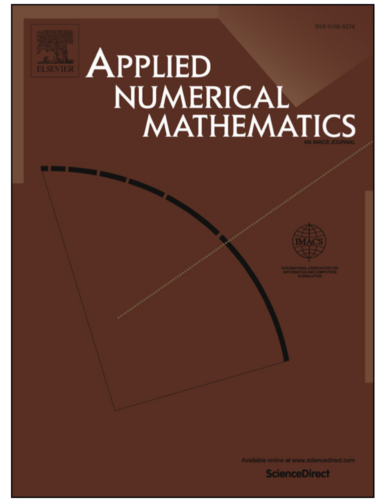
Accepted Manuscript



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PII: S0168-9274(17)30099-5
DOI: http://dx.doi.org/10.1016/j.apnum.2017.04.003
Reference: APNUM 3197

To appear in: *Applied Numerical Mathematics*

Received date: 27 June 2016
Revised date: 22 December 2016
Accepted date: 11 April 2017

Please cite this article in press as: Z. Liu et al., A fast solution technique for finite element discretization of the space-time fractional diffusion equation, *Appl. Numer. Math.* (2017), http://dx.doi.org/10.1016/j.apnum.2017.04.003

A fast solution technique for finite element discretization of the space-time fractional diffusion equation

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Abstract

In this paper, we study fast Galerkin finite element methods to solve a space-time fractional diffusion equation. We develop an optimal piecewise-linear and piecewise-quadratic finite element methods for solving this problem and give optimal error estimates. Furthermore, we develop piecewise-constant discontinuous finite element method for discontinuous problem of this model. Importantly, a fast solution technique to accelerate non-square Toeplitz matrix-vector multiplications which arise from both continuous and discontinuous Galerkin finite element discretization respectively is considered. This fast solution technique is based on fast fourier transform and depends on the special structure of coefficient matrices and it helps to reduce the computational work from $O(N^3)$ required by the traditional methods to $O(Nlog^2N)$, where N is the size (number of spatial grid points) of the coefficient matrices for every time step. Moreover, the applicability and accuracy of the method are demonstrated by numerical experiments to support our theoretical analysis.

Keywords:
Fast Galerkin methods, Space-time fractional diffusion equation, Toeplitz matrix, Fast fourier trasform

1. Introductions

In recent years, many problems in physical science, electromagnetism, electrochemistry, diffusion and general transport theory can be solved by the fractional calculus approach, which gives attractive applications as a new modeling tool in a variety of scientific and engineering fields. Roughly speaking, the fractional models can be classified into two principal kinds: space-fractional differential equation and time-fractional one. Numerical methods and theory of solutions of problems for fractional differential equations have been studied extensively by many researchers which mainly cover finite element methods [1–4], mixed finite element methods [5–8], finite difference methods [9–13], finite volume (element) methods [14, 15], (local) discontinuous Galerkin (L)DG methods [16], spectral methods [17, 18] and so on.

Space-time fractional diffusion equations are used as models for anomalous transport in many disciplines such as hydrogeology, biology, etc. In this paper, we consider the one dimension space-time fractional diffusion equation(see [19])

$$\begin{cases} \dfrac{\partial^\gamma u}{\partial t^\gamma} = -c^2 \mathscr{L} u + f(x,t), \quad x \in \Omega \\ u(x,t) = 0, \qquad x \in \Gamma, \ t \in [0,T], \\ u(x,0) = u_0(x), \quad x \in \Omega, \end{cases} \tag{1}$$

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