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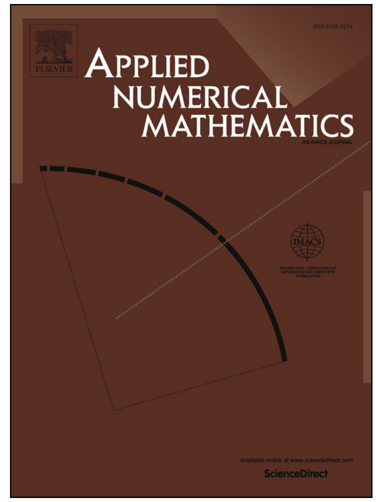

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A second order operator splitting numerical scheme for the "good" Boussinesq equation

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Abstract

The nonlinear stability and convergence analyses are presented for a second order operator splitting scheme applied to the "good" Boussinesq equation, coupled with the Fourier pseudo-spectral approximation in space. Due to the wave equation nature of the model, we have to rewrite it as a system of two equations, for the original variable u and $v = u_t$, respectively. In turn, the second order operator splitting method could be efficiently designed. A careful Taylor expansion indicates the second order truncation error of such a splitting approximation, and a linearized stability analysis for the numerical error function yields the desired convergence estimate in the energy norm. In more details, the convergence in the energy norm leads to an $\ell^\infty(0, T^*; H^2)$ convergence for the numerical solution u and $\ell^\infty(0, T^*; \ell^2)$ convergence for $v = u_t$. And also, the presented convergence is unconditional for the time step in terms of the spatial grid size, in comparison with a severe time step restriction, $\Delta t \le Ch^2$, required in many existing works.

Keywords: "good" Boussinesq equation, operator splitting, Fourier pseudo-spectral method, aliasing error, stability and convergence analysis

AMS subject classification: 65M12, 65M70

1 Introduction

In this article we study a commonly used soliton-producing nonlinear wave equation, the so-called "good" Boussinesq (GB) equation:

$$u_{tt} = -u_{xxxx} + u_{xx} + (u^p)_{xx}, \quad \text{with an integer } p \ge 2. \tag{1.1}$$

In comparison with the well-known Korteweg-de Vries (KdV) equation, the wave equation form of (1.1) introduces the second order temporal derivative. The GB equation and its various extensions have been extensively analyzed in the existing literature, such as a closed form solution for the two soliton interaction in [46], a highly complicated mechanism for the solitary waves interaction in [47], and the nonlinear stability and convergence of some simple finite difference schemes in [49]. Many other analytical and numerical works related to GB equations could also be found, for example, in [1, 7, 8, 14, 15, 24, 25, 37, 48, 50, 57].