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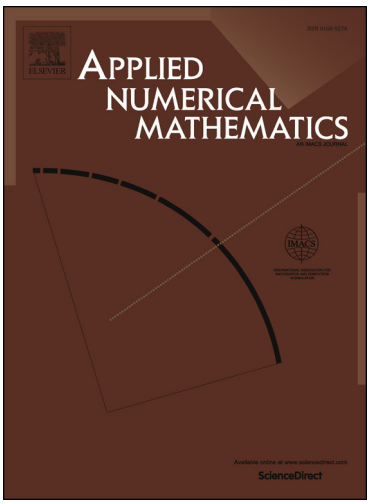

A numerical method for the solution of exterior Neumann problems for the Laplace equation in domains with corners

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A numerical method for the solution of exterior Neumann problems for the Laplace equation in domains with corners

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Abstract

In this paper we propose a new boundary integral method for the numerical solution of Neumann problems for the Laplace equation, posed in exterior planar domains with piecewise smooth boundaries. Using the single layer representation of the potential, the differential problem is reformulated as a classical boundary integral equation. The use of a smoothing transformation and the introduction of a modified Gauss-Legendre quadrature formula for the approximation of the singular integrals, which turns out to be convergent, lead us to apply a Nyström type method for the numerical solution of the integral equation. We solve some test problems and present the numerical results in order to show the efficiency of the proposed procedure.

Keywords: Boundary integral equations, Neumann problem, Nyström method
2000 MSC: 65R20

1. Introduction

In this work we are concerned with the exterior Neumann problem for the Laplace equation in two-dimensional domains with piecewise smooth boundaries. Let $\Omega \subset \mathbb{R}^2$ be an open bounded simply connected domain, with a piecewise smooth Lipschitz boundary Γ. We shall assume that the boundary curve Γ contains n corner points $P_1, \ldots, P_n$ and is otherwise smooth. We consider the problem

$$\begin{aligned} &\Delta u = 0, && \text{in } \mathbb{R}^2 \setminus \bar{\Omega}, \\ &\frac{\partial u}{\partial \mathbf{n}} = f, && \text{on } \Gamma, \\ &|u(x)| = o(1), && \text{as } |x| \to \infty, \end{aligned} \tag{1}$$

where $\mathbf{n}$ denotes the outward unit normal vector at Γ. We shall assume that the given Neumann data f is a sufficiently smooth function which satisfies

$$\int_\Gamma f ds = 0. \tag{2}$$

It is well known that the solution of the exterior problem (1)-(2) exists and is unique (see, for instance, [21, p. 73],[20, p. 351],[5, p. 152]). A boundary integral equation (BIE) formulation of the exterior Neumann problem (1) is obtained by using the single layer representation of the potential u, i.e.

$$u(x) = -\int_\Gamma \phi(y) \log|x-y| dS(y), \quad x \in \mathbb{R}^2 \setminus \bar{\Omega}, \tag{3}$$

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