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Numerical solution of system of Volterra integral equations with weakly singular kernels and its convergence analysis

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ABSTRACT

In this paper, efficient and computationally attractive methods based on the Sinc approximation with the single exponential (SE) and double exponential (DE) transformations for the numerical solution of a system of Volterra integral equations with weakly singular kernels are presented. Simplicity for performing even in the presence of singularities is one of the advantages of Sinc methods. Convergence analysis of the proposed methods is given and an exponential convergence is achieved as well. Numerical results are presented which demonstrate the efficiency and high accuracy of the proposed methods.

1. Introduction

Over the past decades, numerical methods for solving integral equations have attracted continuous interest and have been a subject of a large number of studies [1–4,6–8,11,13,19,23,27,31]. However, not all kinds of the integral equations have received equal attention. Numerical solution of weakly singular integral equations has been rarely considered in the literature. Also, many interesting problems of physics, biomathematics lead to the system of Volterra integral equations. For instance, some heat transfer problems in physics, many models for neural networks in biomathematics, nuclear reactor dynamics problems and thermo-elasticity problems are usually replaced by a system of Volterra integral equations [9,12,20]. In this context, nonlinear systems are more complicated and applicable in real-world problems. So, it is important and favorable to develop a suitable algorithm for them.

The main goal of this paper is to propose efficient methods to approximate the solution of system of linear weakly singular Volterra integral equations, namely

$$U(t) = F(t) + \int_0^t (t-s)^{-\mu} K(t,s) U(s) \mathrm{d}s, \qquad t \in I = [0, T], \tag{1}$$

and also system of nonlinear weakly singular Volterra integral equations,

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$$U(t)=F(t)+\int_0^t (t-s)^{-\mu}\hat{K}(t,s,U(s))\mathrm{d}s, \qquad t\in I=[0,T], \tag{2}$$

where $0<\mu<1$ and

$$K(t,s)=\begin{bmatrix} K_{11}(t,s) & K_{12}(t,s)\\ K_{21}(t,s) & K_{22}(t,s)\end{bmatrix}, \qquad \hat{K}(t,s,U(s))=\left(\hat{K}_1(t,s,U(s)),\hat{K}_2(t,s,U(s))\right)^T,$$
$$U(t)=(u_1,u_2)^T=(x(t),y(t))^T, \qquad F(t)=(g(t),h(t))^T,$$

such that the functions g, h, the kernels $K_{kl}(.,.)$ $(k,l=1,2)$ and $\hat{K}_r(.,.,U(.))$ $(r=1,2)$ are known smooth functions.

The existence and uniqueness results for the solution of systems of integral equations with weakly singular kernels have been given in [5]. System of weakly singular Volterra integral equations is usually difficult to solve analytically so, it is important to find their approximate solutions by applying some numerical methods. More recently, Maleknejad and Salimi Shamloo [17] utilized operational matrices of piecewise constant orthogonal functions to find numerical solutions for system of linear singular Volterra integral equations. The Chebyshev collocation scheme being applied to solve linear weakly singular integral algebraic equations is offered in [24].

With these backgrounds, we propose Sinc-collocation methods for solving linear and nonlinear systems of Volterra integral equations with weakly singular kernels. There has been a growing interest in using Sinc methods. Their wide-spreading use can be returned to a number of benefits. The main advantage of these methods is that they can be utilized directly to the problems without requiring other methods, such as preconditions, to convert the system of equations to a well-conditioned system. Another important advantage is that by using Sinc-collocation methods high accuracy approximate solution can be easily computed.

The Sinc approximation has been employed to solve ordinary and partial differential equations [18,25,26,28] and integral equations [15,16,32] by many authors, e.g., Saadatmandi, Razzaghi, Zakeri, Rashidinia and Maleknejad.

The outline of the paper is as follows: Section 2 is devoted to some preliminaries such as notation, definition, and basic theorems which will be needed further on. In Section 3, we present computational methods for solving system (1) and (2). Convergence analysis of the proposed numerical methods is investigated in Section 4 and an exponential convergence is obtained as well. The final section present some numerical examples with the aim of demonstrating the efficiency, accuracy and convergence behavior of the methods.

2. Preliminaries

In this section, we recall notations and definition of the Sinc function and basic theorems of Sinc methods that are useful for this paper. These are discussed thoroughly in [14,29].

2.1. Functional spaces

Firstly, it is convenient that introduce the following function spaces that clarify the next conditions more precisely.

Definition 1. Let β, γ be a positive constants, and let D be a bounded and simply-connected domain which satisfies $(a,b)\subset D$. Then $L_{\beta,\gamma}(D)$ denotes the family of functions f that satisfy the following conditions:

(i) f is analytic in D;
(ii) there exists a positive constant C such that for all z in D

$$|f(z)|\le C|z-a|^{\beta}|b-z|^{\gamma}. \tag{3}$$

For later convenience, let us denote $L_\beta(D)=L_{\beta,\beta}(D)$ and introduce a function $Q(z)=(z-a)(b-z)$.

It should be pointed out that the family of all functions f that are analytic in domain D is denoted by $Hol(D)$.

Let us introduce another function space $M_\alpha(D)$ in the following definition.

Definition 2. Let α be a constant which satisfies $0<\alpha\le 1$ and let D be a bounded and simply-connected domain such that $(a,b)\subset D$. The space $M_\alpha(D)$ consists of all functions f that satisfy the following conditions:

(i) f is analytic in D and continuous on $\bar{D}$.
(ii) there exists a constant C for all z in D such that

$$|f(z)-f(a)|\le C|z-a|^{\alpha},$$
$$|f(b)-f(z)|\le C|b-z|^{\alpha}.$$